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COMPARISON OF ADDED MASS MODELING FOR SHIPS

by

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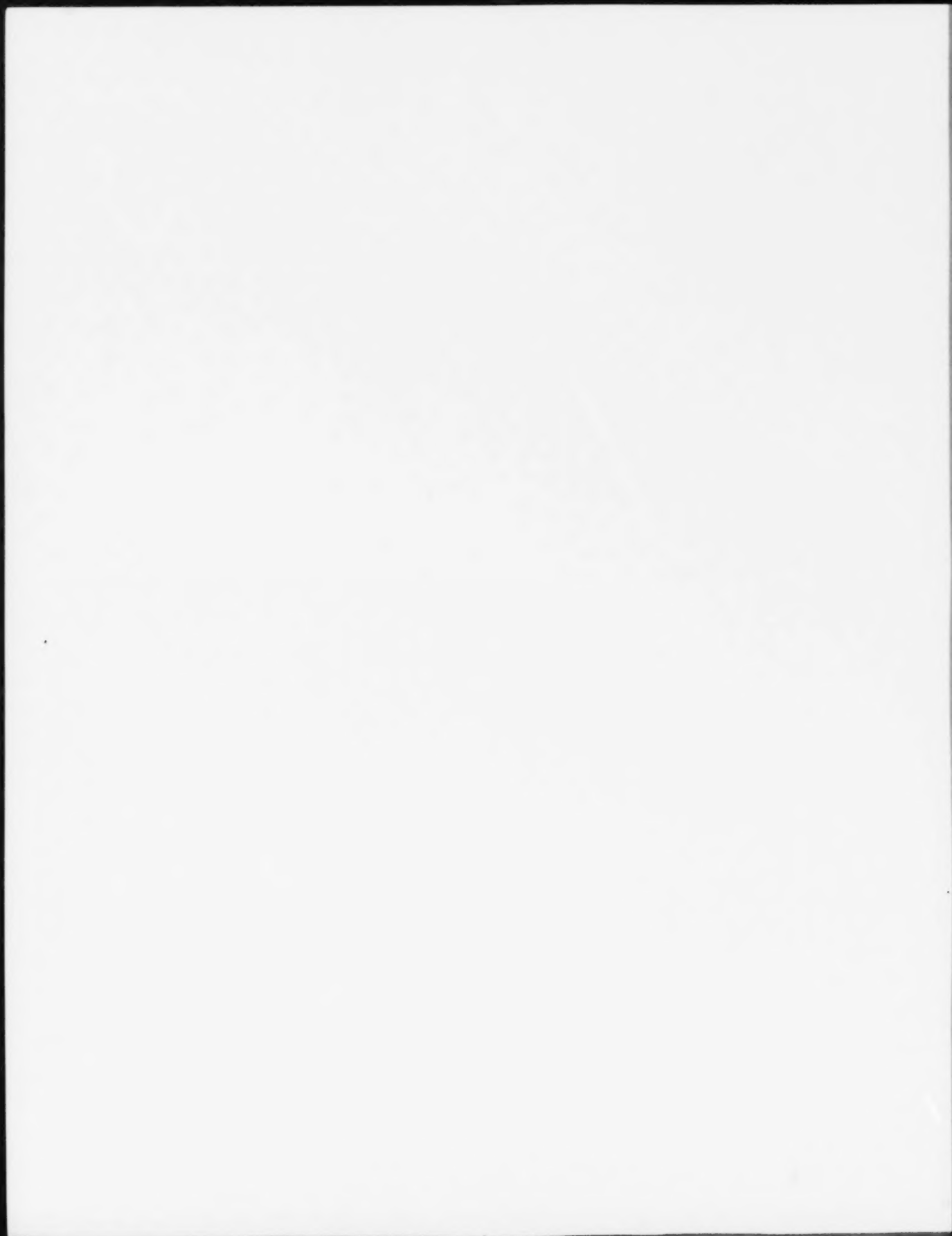
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Abstract

g/ This report presents a comparison of added mass modeling techniques that may be used to determine the vibration response characteristics of ships in water. The mathematical treatment of added mass is reviewed, and a number of numerical approaches are discussed. Examples are presented of the different numerical approaches when applied to a ship model and numerical results for the natural frequencies are compared with experimentally determined values. //

RÉSUMÉ

Le présent rapport traite d'une comparaison des techniques de modélisation de la masse ajoutée qui peuvent être utilisées pour déterminer les caractéristiques de tenue aux vibrations des navires dans l'eau. Le traitement mathématique de la masse ajoutée est étudié et un certain nombre d'approches numériques sont traitées. Des exemples des différentes approches numériques utilisées avec un modèle de navire sont donnés et des résultats numériques des fréquences propres sont comparés à des valeurs obtenues de manière expérimentale.



Contents

Table of Contents	1
List of Tables	3
List of Figures	5
Nomenclature	6
1 Introduction	8
2 Theoretical Background	9
2.1 Strip Theory	9
2.2 Mathematical Treatment of Added Mass	11
2.3 Vibration of a Submerged Beam	15
2.4 Numerical Approaches	19
2.4.1 Finite Element Method	20
2.4.2 Boundary Element Method	25
2.4.3 HYDRO	28
2.4.4 Modal Analysis of Fluid-Structure Vibration	28
2.4.5 Modal Analysis Using Wet - Dry Mode Equivalence . .	30
3 Finite Element Implementation of Added Mass	32
3.1 Wet Mode Category	33
3.2 Dry Mode Category	34
3.2.1 Modal Analysis	34
4 Comparison of Results	38
4.1 Ship Model	38

4.1.1	Experimental Results	38
4.1.2	Finite Element Results	40
4.1.3	Modal Analysis Results	48
4.1.4	PANEL Results	64
4.2	Plate Model	66
4.2.1	Cantilevered Boundary Condition	66
4.2.2	Free-Free Boundary Condition	69
5	Conclusions	72

List of Tables

4.1	Experimental Natural Frequencies (in Hertz) of Ship Model . .	40
4.2	Comparison of Natural Frequencies in Air - VAST	40
4.3	Comparison of CPU Times in Hours for S12-8 Using a VAX 750	41
4.4	Comparison of Natural Frequencies in Water - VAST	44
4.5	Comparison of Δ Frequency in Air and in Water	44
4.6	Comparison of CPU Times in Hours for Ship Model	47
4.7	Comparison of Natural Frequencies in Water - VLS	48
4.8	Comparison of Air Frequencies Between Experimental Model and Simplified Beam Model	53
4.9	Comparison of Air Frequencies Between Glenwright's Experi- mental Model and Simplified Beam Model	53
4.10	Comparison of Natural Frequencies - 14 Mode Approximation, S12-8	56
4.11	Comparison of Natural Frequencies - 1 Mode Approximation, S12-8	56
4.12	Natural Frequencies of S12-8, In Ascending Order	57
4.13	Summary of Degrees of Freedom in Generalized Added Mass for S12-8	60
4.14	Ratio of Natural Frequencies	61
4.15	Ratio of Natural Frequencies Calculated from Generalized Added Mass	62
4.16	Generalized Added Mass Calculated from Mode Shapes in Water	63
4.17	Generalized Added Mass Calculated from Mode Shapes in Air	63
4.18	Comparison of Air Natural Frequencies (Hz) for Ship Model With Uniform Mass Distribution	64
4.19	Comparison of Water Natural Frequencies (Hz) For Ship Model With Uniform Mass Distribution	64

4.20 Comparison of Natural Frequencies - VPS, 1 Mode Approximation	65
4.21 Comparison of CPU Times in Seconds for Cantilevered Plate .	66
4.22 Comparison of CPU Times in Seconds for Free Free Plate . . .	71

List of Figures

2.1	Variation of J Factor for the Fundamental Mode Along the Length of a Circular Beam	18
2.2	Boundary Value Problem	21
3.1	Stages of Evaluation for Ship Vibration Problems	35
4.1	Experimental Ship Model	39
4.2	2 Node Vertical Mode of Ship in Air	42
4.3	3 Node Vertical Mode of Ship in Air	42
4.4	4 Node Vertical Mode of Ship in Air	43
4.5	5 Node Vertical Mode of Ship in Air	43
4.6	2 Node Vertical Mode of Ship in Water	45
4.7	3 Node Vertical Mode of Ship in Water	45
4.8	4 Node Vertical Mode of Ship in Water	46
4.9	5 Node Vertical Mode of Ship in Water	46
4.10	Iterative Solution	49
4.11	Comparison of Mode Shapes, 2 Node Vertical Mode	51
4.12	Comparison of Mode Shapes, 3 Node Vertical Mode	51
4.13	Comparison of Mode Shapes, 4 Node Vertical Mode	52
4.14	Comparison of Mode Shapes, 5 Node Vertical Mode	52
4.15	Comparison of Solution Times for Modal Analysis	55
4.16	Generalized Added Mass for S12-8, $\bar{M}_a$	59
4.17	Plate Model	67
4.18	Cantilevered Plate - Comparison of Results Between Finite Element Analysis and Modal Analysis	68
4.19	Free Free Plate - Comparison of Results Between Finite Element Analysis and Modal Analysis	70

Nomenclature

k	=	Structural Stiffness
m	=	Structural Mass
m_a	=	Added Mass
ω_a	=	Structural Natural Frequency in Air
ω_w	=	Structural Natural Frequency in Water
J	=	Longitudinal Inertia Coefficient
T	=	Kinetic Energy of Fluid per Unit Length
B	=	Half Beam of Ship Section
ρ	=	Density of Fluid
v	=	Sectional Velocity
g	=	Gravitational Constant
C	=	Cross Sectional Inertia Coefficient
p	=	Pressure
$\vec{F}$	=	Force Vector
$\vec{n}$	=	Normal Vector
ϕ	=	Velocity Potential
$\vec{r}$	=	Radius Vector from Center of Rotation
U_1	=	Surge Velocity
U_2	=	Heave Velocity
U_3	=	Sway Velocity
U_4	=	Roll Angular Velocity
U_5	=	Yaw Angular Velocity

U_0	=	Pitch Angular Velocity
m_{ij}	=	i 'th direction Added Mass given unit Acceleration in j 'th direction
A_j	=	Amplitude of Sinusoidal Motion in j 'th direction
a_{ij}	=	Added Mass in the presence of Free Surface
b_{ij}	=	Added Damping in the presence of Free Surface
E	=	Young's Modulus
I	=	Area Moment of Inertia
N_i	=	Shape Functions
$\underline{\underline{M_s}}$	=	Structural Mass Matrix
$\underline{\underline{K_s}}$	=	Structural Stiffness Matrix
$\underline{\underline{C_s}}$	=	Structural Damping Matrix
$\underline{\underline{T}}$	=	Transformation Matrix Relating Nodal Velocity to Centroidal Velocity
$\underline{\underline{C}}$	=	Transformation Matrix Relating Element Centroid Velocity to Element Source Strength
$\underline{\underline{V}}$	=	Transformation Matrix Relating Potential Flux to Source Strength
$\underline{\underline{A}}$	=	Added Mass Matrix Calculated from Boundary Element Method
$\underline{\underline{B}}$	=	Added Damping Matrix Calculated from Boundary Element Method
ϕ_a	=	Natural Mode Shape in Air
ϕ_w	=	Natural Mode Shape in Water

Chapter 1

Introduction

The vibration characteristics of a structure are significantly affected by the medium in which the structure is vibrating. Thus the vibration characteristics of a ship in water cannot be deduced without due attention being paid to the modeling of the added mass effects of the surrounding fluid. From a designer's view point, it is important to have accurate estimates of the fundamental hull modes of a ship at an early stage in order to avoid resonant excitation caused by propeller forces or by wave interaction.

Various techniques have been developed for the analysis of fluid effects amongst which the finite element and boundary element methods have been the most popular. In this report, the fundamental theory governing the interaction between the fluid and a floating structure is reviewed and a discussion is presented of the various numerical procedures that are available to solve the problem. Recommendations are presented as to the most efficient procedure for analysing the effect of the fluid.

Chapter 2

Theoretical Background

In this section, a review is presented of the various mathematical approaches that have been developed in order to estimate the effect of the fluid on the vibration response of a submerged and floating structure.

2.1 Strip Theory

In 1927, Frank Lewis [25], submitted a paper to the Society of Naval Architects and Marine Engineers which outlined procedures for calculating the inertia of the water surrounding a vibrating ship. He recognized the relationship between natural frequency of a vibrating ship and its stiffness and mass. According to Lewis,

"... the added mass can be evaluated with fair accuracy by the methods explained in this paper. Accurate knowledge of a ship's stiffness is still lacking. It is recognized that the flexural rigidity of a hull is somewhat less than that based on the nominal I of its cross-section... Until further experiments on hull rigidity have been made, the frequency calculations must rest upon an empirical basis..."

Lewis solved the added mass problem in two steps. First, he realized that fluid flow surrounding an arbitrary vibrating ship is complex. The fluid dynamics problem would be much easier to solve if it were two dimensional. Thus a relationship between two dimensional flow and three dimensional flow

must be established. Next, he noted that added mass for ships with different cross sections is different. Hence a relationship between added mass and ship cross section must also be established.

Among the few three dimensional problems in fluid dynamics for which an exact solution has been obtained is that of flow around an ellipse of revolution completely immersed in an infinite fluid. From the exact solution of this problem, Lewis derived a formula for longitudinal J factors corresponding to the 2 node and 3 node vertical modes. These factors are defined as

$$J = \frac{\text{longitudinal inertia coefficient}}{\text{actual K.E. of surrounding fluid}} = \frac{\text{K.E. of surrounding fluid if motion is confined to transverse plane}}{\text{actual K.E. of surrounding fluid}}$$

That is, J factors indicate the ratio of kinetic energies between three dimensional flow and two dimensional flow. Lewis assumed that J factors are the same for ships with similar length to beam ratios. Thus, fluid flow surrounding any vibrating ship has been reduced to a two dimensional problem.

Traditionally, two dimensional fluid dynamic problems are solved by conformal mapping. The kinetic energy per unit length of fluid surrounding a cylinder of circular cross section moving as a rigid body is given by

$$2T = \pi B^2 \rho v^2$$

where

- T = kinetic energy
- B = half beam perpendicular to the motion
- ρ = density of the fluid
- v = velocity
- g = gravity constant.

Then, for all other cross sections with the same beam, the energy can be written as

$$2T = C \pi B^2 \rho v^2$$

where C is the inertia coefficient for that cross section. Thus the additional inertia mass per unit length of any ship cross section can be written as

$$m_a = \frac{\rho}{2} C J \pi B^2$$

Recall that the J factor is included to account for three dimensional flow.

Lewis' work arose from the inability of researchers to calculate fluid effects surrounding a vibrating ship. J factors were assumed to be the same for ships of similar length to beam ratios. They were calculated only for the 2 node and 3 node vertical modes. The free surface effects have been neglected. An arbitrary ship cross section is transformed to a circular cross section by a series of conformal transformations, from which the sectional inertia coefficient C is found.

2.2 Mathematical Treatment of Added Mass

With some restrictions, the added mass of a submerged body can be calculated mathematically. Newman [27] derived the rigid body added mass of a structure vibrating in an infinite fluid as well as on a free surface. This derivation differs in many respects to the work by Lewis.

- The added mass is derived in integral form.
- Sectional inertia coefficients are not required.
- The free surface is included in the derivation.

Added Mass For A Three Dimensional Body In An Infinite Fluid

Consider a body with surface Γ_b surrounded by a fluid of volume V , having an outer imaginary surface Γ_o . The forces acting on the body are represented by the integral of the pressure over the surface of the body, or

$$\vec{F} = \iint_{\Gamma_b} p \vec{n} d\Gamma, \quad (2.1)$$

- where $\vec{F}$ = the sum of forces acting on the body,
 p = the pressure of the fluid over the body, and
 $\vec{n}$ = the normal vector pointing out of the fluid.

Substituting Bernoulli's equation,

$$\frac{p}{\rho} + \left\{ \frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi \right\} = 0 \quad (2.2)$$

into (2.1), the force on the body is given by

$$\vec{F} = - \iint_{\Gamma_b} \rho \left\{ \frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi \right\} \vec{n} d\Gamma \quad (2.3)$$

Using the Transport theorem,

$$\frac{d}{dt} \iiint_V f(\vec{x}, t) dV = \iiint_V \frac{\partial f}{\partial t} dV + \iint_S f(\vec{U} \cdot \vec{n}) dS \quad (2.4)$$

Gauss' theorem,

$$\iiint_V \nabla f dV = \iint_S f \vec{n} dS \quad (2.5)$$

and by assuming that the body is moving in an ideal infinite fluid, equation 2.3 can be reduced to

$$\vec{F} = -\rho \frac{d}{dt} \iint_{\Gamma_b} \phi \vec{n} d\Gamma \quad (2.6)$$

If the body is moving (with the motion having six degrees of freedom), the velocity potential must satisfy the boundary condition Γ_b ,

$$\frac{\partial \phi}{\partial n} = \vec{U} \cdot \vec{n} + \vec{\Omega} \cdot (\vec{r} \times \vec{n}) \quad (2.7)$$

where $\vec{U}$ is the translational velocity denoted by (U_1, U_2, U_3) , $\vec{\Omega}$ is the angular velocity of the rotating body by (U_4, U_5, U_6) , and $\vec{r}$ is the radius vector from the center of rotation. In Naval Architecture terms, $(U_1, U_2, U_3, U_4, U_5, U_6)$ are identified as (surge, heave, sway, roll, yaw, and pitch). The boundary condition 2.7 suggests that the velocity potential can be written as a sum of six components, or

$$\phi = \sum_{i=1}^6 U_i \phi_i, \text{ and} \quad (2.8)$$

$$\frac{\partial \phi}{\partial n} = \sum_{i=1}^6 U_i \frac{\partial \phi_i}{\partial n} \quad (2.9)$$

Using vector analysis, and substituting equations 2.8 and 2.9, equation 2.6 can be reduced to

$$F_j = -\rho \sum_{i=1}^6 \dot{U}_i(t) \iint_{\Gamma_b} \phi_i \frac{\partial \phi_j}{\partial n} d\Gamma \quad (2.10)$$

Or,

$$F_j = \sum_{i=1}^6 m_{ij} \dot{U}_i(t) \quad (2.11)$$

where

$$m_{ij} = \rho \iint_{\Gamma_b} \phi_i \frac{\partial \phi_j}{\partial n} d\Gamma \quad (2.12)$$

is the added mass associated with the body moving in an infinite fluid.

A property of the added mass matrix is that it is symmetric. That is

$$m_{ij} = m_{ji}. \quad (2.13)$$

This property can be easily shown by Green's second identity,

$$\iiint (\psi \nabla^2 \phi - \phi \nabla^2 \psi) d\Delta = \iint (\psi \frac{\partial \phi}{\partial n} - \phi \frac{\partial \psi}{\partial n}) d\Gamma. \quad (2.14)$$

If both ψ and ϕ are potential functions, then the left hand side vanishes. That is

$$\iint (\psi \frac{\partial \phi}{\partial n} - \phi \frac{\partial \psi}{\partial n}) d\Gamma = 0. \quad (2.15)$$

Or,

$$\iint \psi \frac{\partial \phi}{\partial n} d\Gamma = \iint \phi \frac{\partial \psi}{\partial n} d\Gamma. \quad (2.16)$$

Added Mass and Damping in a Free Surface Application

The added mass coefficients m_{ij} of a body moving in an infinite ideal fluid were derived in the previous section. Derivation of added mass and damping in the more complicated situation of a body moving on a free surface follow the same approach.

The motion of the body on or near a free surface is defined to be,

$$U_j(t) = \text{Re}\{i\omega A_j e^{i\omega t}\} \quad (2.17)$$

where $j = 1, 2, \dots, 6$ represents the six motions (surge, heave, sway, roll, yaw, and pitch) of the body and A_j is the amplitude of the sinusoidal motion.

Ignoring the presence of incident waves the velocity potential can be written as

$$\phi(x, y, z, t) = \operatorname{Re}\left\{\sum_{j=1}^6 A_j \phi_j(x, y, z) e^{i\omega t}\right\} \quad (2.18)$$

For example, if a body were forced to oscillate in heave motion with unit amplitude, in calm water, the resulting fluid motion can be represented by the velocity potential ϕ_3 .

The pressure over the body, ignoring second order terms, is given as

$$p = -\rho\left\{\frac{\partial \phi}{\partial t} + gy\right\}$$

or,

$$p = -\rho \operatorname{Re}\left\{\sum_{j=1}^6 A_j \phi_j i\omega e^{i\omega t}\right\} - \rho gy \quad (2.19)$$

Substituting the pressure into equation 2.1, the forces and moments on the body are

$$\begin{pmatrix} \vec{F} \\ \vec{M} \end{pmatrix} = -\rho g \iint_{S_b} \begin{pmatrix} \vec{n} \\ \vec{r} \times \vec{n} \end{pmatrix} y \, dS - \rho \iint_{S_b} \operatorname{Re} \sum_{j=1}^6 i\omega A_j e^{i\omega t} \begin{pmatrix} \vec{n} \\ \vec{r} \times \vec{n} \end{pmatrix} \phi_j \, dS \quad (2.20)$$

Note that equation 2.20 contains two terms: the first being a hydrostatic restoring force and the second representing a force similar to that derived from equation 2.11. But instead of a simple added mass as before, we now have real and imaginary terms because the velocity potential in this problem is complex.

Thus the six components of force and moment on the body can be written as,

$$F_i = \operatorname{Re}\left\{\sum_{j=1}^6 A_j e^{i\omega t} f_{ij}\right\}; \quad i = 1, 2, \dots, 6 \quad (2.21)$$

$$\text{where } \begin{cases} f_{ij} = -\rho \iint_{\Gamma_b} \frac{\partial \phi_i}{\partial n} \phi_j \, d\Gamma \\ \frac{\partial \phi_i}{\partial n} = i\omega n_i & i = 1, 2, 3 \\ \frac{\partial \phi_i}{\partial n} = (\vec{r} \times \vec{n})_{i-3} & i = 4, 5, 6. \end{cases}$$

The coefficient f_{ij} is physically interpreted as the complex force in the direction i , due to a sinusoidal motion of unit amplitude in the direction j . The added mass and damping coefficients are given by,

$$f_{ij} = \omega^2 a_{ij} - i\omega b_{ij} \quad (2.22)$$

Substituting (2.22) into (2.21) yields,

$$\begin{aligned} F_i &= \operatorname{Re}\left\{\sum_{j=1}^6 A_j e^{i\omega t} (\omega^2 a_{ij} - i\omega b_{ij})\right\} \\ &= \operatorname{Re}\left\{\sum_{j=1}^6 A_j e^{i\omega t} \omega^2 a_{ij} - A_j e^{i\omega t} i\omega b_{ij}\right\} \\ &= -\sum_{j=1}^6 \{\dot{U}_j a_{ij} + U_j b_{ij}\} \end{aligned} \quad (2.23)$$

The coefficient a_{ij} is known as the added mass since it represents the force component proportional to the acceleration of the body. Similarly, the coefficient b_{ij} is known as the added damping; it represents the force component proportional to the velocity of the body.

Note that added mass a_{ij} calculated from this section is different in *value* than m_{ij} from the previous section. As the vibration frequency approaches 0,

$$a_{ij} \rightarrow \frac{1}{2} m_{ij} \quad (2.24)$$

for horizontal plane motion (ie sway, surge, and yaw). Equation 2.24 is also valid for heave, roll, and pitch as the vibration frequency approaches infinity. Readers are referred to Newman's textbook [27] which offers a concise, physical explanation of equation 2.24.

2.3 Vibration of a Submerged Beam

The previous methods described how fluid surrounding a vibrating ship can be quantified as an added mass; however, neither explain specifically *how* natural frequencies can be predicted from the added mass. J.C. Daidola [7] presented a mathematical technique for the prediction of vertical and lateral natural vibrations of an Euler beam in a fluid. This technique was based

on the simultaneous solution of the mechanical equations of motion for a vibrating beam and the coupled three-dimensional equations of motion of the surrounding fluid. He also investigated the applicability of the "J-factor" approach in considering the effects of the fluid and gave specific results for circular Euler beams with different boundary conditions.

For a beam vibrating in the y direction, the Euler equation for a uniform beam is given by

$$EI \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = F(x, t) \quad (2.25)$$

where

- x = distance along the length of the beam
- t = time
- $y(x, t)$ = vertical or lateral deflection of the beam
- E = modulus of elasticity of beam
- I = moment of inertia of beam
- $F(x, t)$ = any external force acting on the beam
- L = length of beam
- m = mass per unit length of beam

When a beam is vibrating in a fluid, there are significant forces $F(x, t)$ acting on the beam due to the fluid. $F(x, t)$ can be determined from the linearized Bernoulli equation

$$P(x, y, z, t) = -\rho \frac{\partial \phi}{\partial t} \quad (2.26)$$

which defines the pressure P in terms of the velocity potential ϕ . Hence for a beam with a circular cross section, the force per unit length exerted on the beam by the fluid in the y direction can be written as

$$F(x, t) = \int_0^\infty \rho \frac{\partial \phi}{\partial t} R \sin \theta \, d\theta \quad (2.27)$$

where $\int_0^\infty$ indicates integration around the contour of a cross section of the beam. Using a series of Fourier Transformations and Inverse Fourier Transforms, the governing equation becomes

$$EI \frac{\partial^4 y}{\partial x^4} - \omega^2 m y = -\frac{\rho R \omega^2}{2} \cdot \int_{-\infty}^{\infty} \frac{K_1(|p|R) e^{ipz}}{|p|(K_0(|p|R) + K_2(|p|R))} \, dz$$

$$\int_{-\infty}^{\infty} y(x') e^{-ipx'} dx' dp \quad (2.28)$$

where K_0 , K_1 , and K_2 represent Bessel functions and p and x' represent dummy integration variables. Note that the right hand side of equation 2.28 represents the hydrodynamic force on the structure. The solution to equation 2.28, from which the mode shapes and natural frequencies of the beam in water can be found, is outlined in Daidola's thesis [7].

Daidola's approach to solve equation 2.28 is not important to this report. However, some insight may be gained from his results:

1. The results show that the historical approach of using Lewis added masses and J-factors to predict free vibration of beams in fluid gives reasonable results. The hydrodynamic force acting on the beam is given by

$$F(x) = -\frac{\rho R \omega^2}{2} \int_{-\infty}^{\infty} \frac{K_1(|p|R) e^{ipx}}{|p|(K_0 + K_2)} \int_{-\infty}^{\infty} y(x') e^{-ipx'} dx' dp \quad (2.29)$$

Hence the added mass can be written as

$$m_a(x) = -\frac{\rho R}{2y(x)} \int_{-\infty}^{\infty} \frac{K_1(|p|R) e^{ipx}}{|p|(K_0 + K_2)} \int_{-\infty}^{\infty} y(x') e^{-ipx'} dx' dp \quad (2.30)$$

and the local J factor is defined as

$$\begin{aligned} J(x) &= \frac{m_a(x)}{m_{2-D}(x)} \\ &= \frac{m_a(x)}{\frac{1}{2} \rho \pi R^2} \\ &= \frac{-1}{\pi R y(x)} \int_{-\infty}^{\infty} \frac{K_1(|p|R) e^{ipx}}{|p|(K_0 + K_2)} \int_{-\infty}^{\infty} y(x') e^{-ipx'} dx' dp \end{aligned} \quad (2.31)$$

Figure 2.1 from Daidola [7] shows the variation of the J factor along the length of the beam for the fundamental mode of a circular beam with free ends. This contrasts with the traditional approach of placing masses uniformly along the length of a ship of uniform cross section.

2. Natural mode shapes predicted by this approach show that they are not altered by the presence of a fluid. Daidola noted:

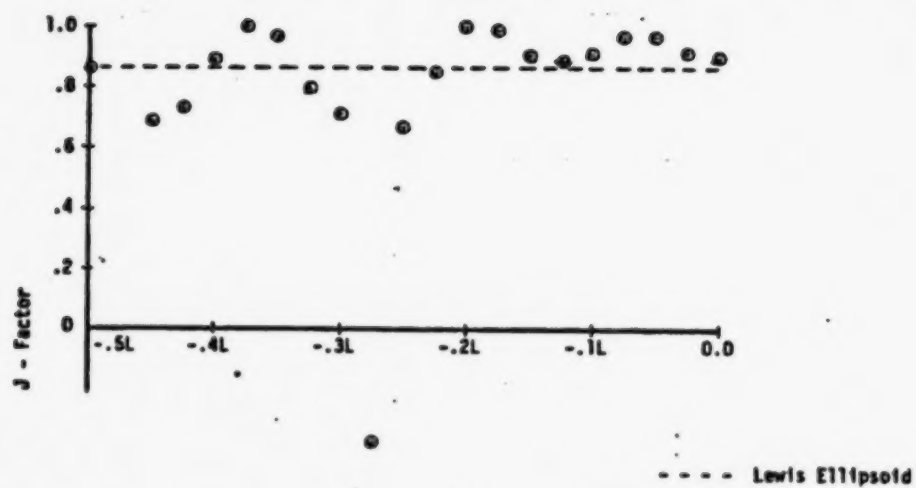


Figure 2.1: Variation of J Factor for the Fundamental Mode Along the Length of a Circular Beam

"... the natural mode shapes of a circular uniform beam do not appear to be altered by the presence of the fluid, regardless of its density ..."

It is interesting to note that even though Daidola first approached the problem by *not* assuming equivalence of wet and dry modes of vibrations, he concluded that

"... Utilization of the in-vacuum mode shapes in modal analysis to determine beam response appears to be justifiable by virtue of the equivalence of the fluid mode shapes... "

However, it should be noted that the analysis used assumed a uniform mass distribution.

2.4 Numerical Approaches

Ship vibration problems are complex. Unless simplifying assumptions are made (restricting the solution to an Euler beam for example, as Daidola has done) closed form solutions usually are not possible. Thus the problems are generally solved with numerical techniques such as finite element or boundary element methods.

The application of finite elements in the investigation of added mass vibration phenomena is well documented. It has been shown [2], [12], [15], [26] that numerical results are consistent with experimental results. The main drawback of the finite element method is the computation time required to solve a large problem.

Implementation of the boundary element method is well suited to the evaluation of the added mass matrix of a vibrating ship. The effect of the fluid on the ship can be represented by a distribution of source singularities along the bounding surface, and a reduction in the number of variables is achieved because the governing equations have been reduced by a dimension. This approach is attractive in view of the computation time saved as compared to the finite element method.

2.4.1 Finite Element Method

There have been many papers [2], [12], [15], [26], [28] written about using finite element methods to calculate the added mass of a vibrating structure.

The fluid-structure interaction problem can be formulated as a boundary value problem as represented by figure 2.2 where the pressure p is to satisfy

$$\nabla^2 p = 0 \quad (2.32)$$

inside the domain. At every point on the fluid-structure interface, S_h ,

$$\frac{\partial p}{\partial n} = -\rho \ddot{u}_n \quad \text{on } S_h \quad (2.33)$$

where u_n is the displacement normal to the element. The equilibrium condition at the free surface is generally used as

$$p = 0 \quad \text{on } S_f \quad (2.34)$$

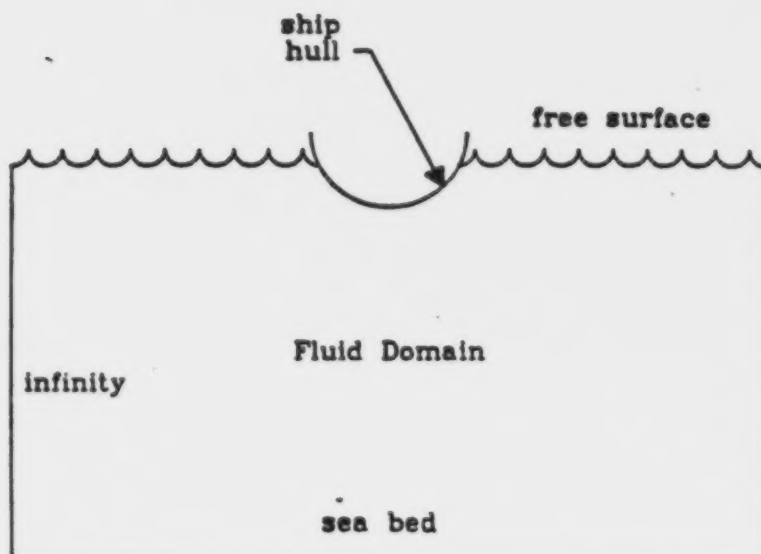
This is a valid assumption in the frequency range of interest for propeller induced vibrations (6 to 15Hz), but is not valid at very low frequencies. For example rigid-body motions and sometimes even the first fundamental modes of vibration occur at frequencies below 1 Hz for some large ships. In such cases, the linearized free surface boundary condition

$$\frac{\partial p}{\partial y} + \frac{1}{g} \ddot{p} = 0 \quad (2.35)$$

(which is frequency dependent) should be used. For the infinite boundary condition, S_r , a suitable boundary condition must be imposed so that no waves are reflected; since waves generated by the vibrating hull will travel outward and die out at infinity. It was stated [2] that this is equivalent to

$$\frac{\partial p}{\partial n} = 0 \quad \text{on } S_r \quad (2.36)$$

Note that this condition will have to be tested to see that if the boundary has been put back far enough. Generally, the test for this boundary condition involves creating several fluid meshes and determining (from numerical results) if an increase of the outer boundary has a significant impact on the results.



$$\nabla^2 p = 0 \quad \text{in the fluid domain}$$

$$\frac{\partial p}{\partial n} = -\rho \ddot{u}_n \quad \text{fluid structure interface}$$

$$\frac{\partial p}{\partial n} = 0 \quad \text{at infinity}$$

$$p = 0 \quad \left\{ \begin{array}{l} \text{on the free surface} \\ \text{on sea bed} \end{array} \right.$$

Figure 2.2: Boundary Value Problem

This problem may also be formulated in terms of the velocity potential ϕ . Readers are referred to Zienkiewicz [43] for a complete derivation of the formulae. The velocity field $\vec{v}$ in an ideal fluid can be written as

$$\vec{v} = -\nabla\phi \quad (2.37)$$

or

$$\frac{\partial \vec{v}}{\partial t} = -\nabla \dot{\phi} \quad (2.38)$$

Similarly, the pressure is found from

$$\vec{v} = -\frac{1}{\rho} \nabla p \quad (2.39)$$

Equations 2.38 and 2.39 imply

$$-\nabla \dot{\phi} = -\frac{1}{\rho} \nabla p \quad (2.40)$$

Hence the relationship between pressure p and velocity potential ϕ is then given as

$$p = \rho \dot{\phi} \quad (2.41)$$

The finite element formulation follows the classical Galerkin approach. The pressure inside the domain is approximated by known shape functions $N_m(x, y, z)$ such that

$$\begin{aligned} p^*(x, y, z, t) &= N^T(x, y, z) P(t) \\ &= \sum_{i=1}^M N_i(x, y, z) p_i(t) \end{aligned} \quad (2.42)$$

The equilibrium condition is then approximated over the domain, thus leaving a residual R^* where

$$\nabla^2 p^* = R^* \quad (2.43)$$

which, according to the Galerkin procedure is to be made as small as possible. This is accomplished by multiplying the residual by all the shape functions, integrating through the domain D , and setting the result to 0. Thus,

$$\iiint_D N_i \nabla^2 p^* dx dy dz = 0, \quad i = 1, 2, \dots, M \quad (2.44)$$

Integrating the above by parts, and taking into account the boundary conditions, we find (for $m = 1, 2, \dots, M$)

$$\iint_{sh} N_i \frac{\partial p^*}{\partial n} dS - \iiint_D \left[\frac{\partial N_i}{\partial x} \frac{\partial p^*}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial p^*}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial p^*}{\partial z} \right] dx dy dz = 0 \quad (2.45)$$

But

$$\frac{\partial p^*}{\partial x} = \sum_{i=1}^M \frac{\partial N_i}{\partial x} p_i$$

Substituting to get

$$\iint_{sh} N_i \frac{\partial p^*}{\partial n} dS - \underline{H} \underline{P} = 0 \quad (2.46)$$

where $\underline{H}$ over each finite element is given by

$$H_{ij} = \iiint_e \left[\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right] dx dy dz$$

Next the hull displacements are approximated with a different set of shape functions N' so that

$$u_n = N'^T \underline{U}_n$$

and thus

$$\begin{aligned} \frac{\partial p}{\partial n} &= -\rho \tilde{u}_n \\ &= -\rho \sum_{j=1}^M N'_j \tilde{U}_{n,j} \end{aligned}$$

Substituting to get

$$\underline{H} \underline{P} = \underline{B} \tilde{\underline{U}}_n \quad (2.47)$$

where

$$\begin{aligned} H_{ij} &= \iiint_e \left[\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right] dx dy dz \quad \text{and} \\ B_{ij} &= \rho \iint_e N_i N'_j dS \end{aligned}$$

Note that the above equation can be partitioned into

$$\begin{bmatrix} H_{ss} & H_{sf} \\ H_{fs} & H_{ff} \end{bmatrix} \begin{Bmatrix} P_s \\ P_f \end{Bmatrix} = \begin{Bmatrix} 0 \\ B \tilde{U}_n \end{Bmatrix}$$

where the subscript f represent those structural degrees of freedom in contact with the fluid and the subscript s represent those degrees of freedom not in contact with the fluid. Solving first for P_f ,

$$P_s = -H_{ss}^{-1} H_{sf} P_f$$

Thus, we arrive at

$$\begin{aligned} H_f P_f &= B \ddot{U}_n \text{ where} \\ H_f &= H_{ff} - H_{fs} H_{ss}^{-1} H_{sf} \end{aligned} \quad (2.48)$$

Next, we transform the normal acceleration U_n to cartesian coordinates as

$$\ddot{U}_n = \Delta \ddot{U}_f \quad (2.49)$$

where Δ is a rectangular array of the normal coordinate direction cosines and $\ddot{U}_f$ is a vector of cartesian coordinate accelerations of all nodes on the hull surface in contact with the water.

Hence

$$H_f P_f = B \Delta \ddot{U}_f$$

or

$$P_f = H_f^{-1} B \Delta \ddot{U}_f \quad (2.50)$$

Using a standard finite element formulation, the equations of motion for the structure are

$$M_s \ddot{U}(t) + C_s \dot{U}(t) + K_s U(t) = F_s + R_{fs}(t) \quad (2.51)$$

where M_s is the mass matrix of the structure with *no water*, C_s is the damping matrix, K_s is the stiffness matrix, F_s is the vector of external excitation, and R_{fs} is the vector of hydrodynamic forces acting on the immersed part of the hull. The hydrodynamic force, R_{fs} , is found to be

$$R_{fs} = -\frac{1}{\rho} \Delta^T B^T P_f$$

Substituting, the governing equations can be reduced to

$$(M_s + M_a) \ddot{U}(t) + C_s \dot{U}(t) + K_s U(t) = F_s \quad (2.52)$$

where the hydrodynamic mass matrix is given by

$$\underline{\underline{M}}_a = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\rho} \underline{\underline{\Delta}}^T \underline{\underline{B}}^T \underline{\underline{H}}_f^{-1} \underline{\underline{B}} \underline{\underline{\Delta}} \end{bmatrix}$$

The hydrodynamic mass matrix $\underline{\underline{M}}_a$ in this formulation will be a full symmetric matrix of dimension $3S$ by $3\bar{S}$, where S is the number of submerged nodes. Entries of $\underline{\underline{M}}_a$ are simply added to the entries of $\underline{\underline{M}}_s$ to form the global mass matrix. Eigenvalue solution time of the added mass vibration problem will be similar to the structural dynamics problem if $S \ll N$, where N is the total number of structural nodes. However, if S is of the same order of N then solution time is expected to increase dramatically. For example, experience in analysing a model where $N = 574$ and $S = 147$ have shown that the added mass problem took 143.39 hours (22.82 of which was used to formulate the added mass matrix), compared to 11.08 hours for the structural free vibration problem alone.

2.4.2 Boundary Element Method

The added mass of a vibrating ship can also be calculated by the boundary element method. This method is based on Green's second identity

$$\iiint (\psi \nabla^2 \phi - \phi \nabla^2 \psi) d\Delta = \iint (\psi \frac{\partial \phi}{\partial n} - \phi \frac{\partial \psi}{\partial n}) d\Gamma, \quad (2.53)$$

There have been relatively few boundary element results for added mass problems published in the literature. A paper by Vernon [35] (which was based upon the formulation of Vorus [37]) included comparisons to both theoretical and experimental values. In general, results showed good agreement. Vernon concluded that

"... the CPU time ratios for the finite element method versus the panel (boundary element) method varied from a minimum of approximately 10 in the propellor analysis to a maximum of 70 for the floating cylinder ... in general, a CPU time reduction of at least an order of magnitude can be expected using the panel method ..."

A matrix formulation routine, using Green's function (or source distribution method), is outlined below for the added mass of a vibrating ship. The resulting added mass matrix is independent of any predefined vibratory mode shapes and is superimposed directly in the global mass matrix.

The vector $\delta(t)$ is defined as the set of unknown displacements of nodal points on the wetted hull surface. These nodal points must be assumed to be taken from the finite element mesh originally defined by the user. The matrix $\underline{T}$ is defined as the transformation matrix which expresses the normal velocities at the centroids of the elements in terms of the unknown nodal velocities, that is

$$\underline{v} = \underline{T} \dot{\underline{\delta}} \quad (2.54)$$

The matrix $\underline{C}$ is defined as the transformation matrix which relates the normal velocity of an element v_j to the source strengths σ_k over all the elements, or

$$\underline{v} = \underline{C} \underline{\sigma} \quad (2.55)$$

The pressure p_j over element j is given in terms of σ_j and $\dot{\sigma}_j$ as

$$p = -\rho \underline{P} \dot{\underline{\sigma}} - \rho U \underline{V} \underline{\sigma} \quad (2.56)$$

where

$$P_{ij} = \iint_{\Delta S_j} G_{ij} dS$$

is the potential at element i due to unit source density of element j , U is the ship forward speed, and $\underline{V}$ is the transformation matrix which relates the potential flux to the source strength; that is, $\phi_n = \underline{V} \underline{\sigma}$. Next, let the transformation matrix $[\underline{S}]$ be defined as one which relates the force on the element to the pressure on the element

$$\underline{f} = \underline{S} p \quad (2.57)$$

And finally, we find the force on the structure as

$$\underline{f} = -\rho \underline{S} \underline{P} \underline{C}^{-1} \underline{T} \dot{\underline{\delta}} - \rho U \underline{S} \underline{V} \underline{C}^{-1} \underline{T} \underline{\delta} \quad (2.58)$$

Or,

$$\underline{f} = -\underline{A} \dot{\underline{\delta}} - \underline{B} \underline{\delta}$$

where the added mass and damping matrices are given by

$$\underline{A} = \rho \underline{S} \underline{P} \underline{Q}^{-1} \underline{T}$$

$$\underline{B} = \rho U \underline{S} \underline{V} \underline{Q}^{-1} \underline{T} \quad (2.59)$$

$$(2.60)$$

Several important points are inherent in the above formulation.

1. The matrix $\underline{P}$ was derived from the assumption of zero dynamic pressure (and therefore zero velocity potential) on the free surface, which requires that $\omega^2 \gg g$.
2. Matrices $\underline{A}$ and $\underline{B}$ the added mass and damping matrices to be attached to the structural nodes which are in contact with the fluid. Unlike the added mass matrix derived from finite elements, these are generally non-symmetric. Entries of each of the matrices can be physically interpreted as the influence of a unit load on one degree of freedom on another d.o.f. For example, for a certain arrangement of the finite element mesh, the entry corresponding to the first row and the first column of the added mass matrix represent the force on the x 'th d.o.f. of node 1, given a unit acceleration of the x 'th d.o.f. of node 1. Similarly, the entry corresponding to the first row and the fifth column represent the force on the x 'th d.o.f. of node 1 given a unit acceleration of the y 'th d.o.f. of node 2. Hence for a certain numbering scheme, the sum of the first row of every third entry represents the force on the x 'th d.o.f. of node 1 given a rigid body acceleration of the entire structure.
3. The derivation of $\underline{Q}$ and $\underline{V}$ is outlined in the Douglas report [14].
4. Comparison of the Vorus' solutions [37] with those done by F.M. Lewis for estimating the added mass distribution of a vertically vibrating ship showed remarkable agreement. The disadvantages of Lewis' method remain. But, where the vibratory modes of interest are vertical, with mode shapes that can be approximated as one-dimensional, it appears that Lewis' method should provide acceptable engineering accuracy.
5. This formulation is capable of evaluating the added mass matrices of complex multidimensional modal patterns. Assumption of specific vibratory modes is not necessary.

6. Computation time for the added mass and damping matrices is expected to be small because we are working with *surface* integrals instead of *volume* integrals (as in the case of finite elements). For example, computation time of $\underline{A}$ for a large ship model is of the order of 15 minutes interactive time whereas it takes 22.82 hours for the finite element method.

2.4.3 HYDRO

HYDRO is a computer program written at the University of British Columbia which determines the added mass of a two dimensional section, including the free surface effects. The theory behind HYDRO is explained in the report by Hutton, Glenwright, et al. [10]. HYDRO formulates the following boundary value problem:

$$\begin{array}{ll} \nabla^2 \phi = 0, & \text{inside the domain;} \\ \frac{\partial \phi}{\partial n} = V_n, & \text{on the fluid-structure interface} \\ \frac{\partial \phi}{\partial n} - \frac{\omega^2}{g} \phi = 0, & \text{on the free surface;} \\ \lim_{R \rightarrow \infty} R^{\frac{1}{2}} \left\{ \frac{\partial \phi}{\partial R} - i\lambda \phi \right\} = 0, & \text{on the radiation boundary; and} \\ \lim_{y \rightarrow \infty} \frac{\partial \phi}{\partial n} = 0, & \text{on the sea bed.} \end{array}$$

and solves for the velocity potential ϕ . The added mass is then found from equation 2.12. Note that the added mass calculated by HYDRO is a function of vibration frequency because of the frequency dependent linearized free surface boundary condition.

2.4.4 Modal Analysis of Fluid-Structure Vibration

This approach uses the principle of modal superposition to calculate fluid-structure dynamic characteristics and is the basis of the paper by Ohta [28].

The simplest method of incorporating vertical added mass into a finite element model is to find a lumped value representative of the fluid and put it on the diagonal entries of the global mass matrix. Hence, instead of $\underline{M}\ddot{\underline{x}} + \underline{K}\underline{x} = 0$, the following is solved

$$\left[\begin{array}{c|c} M_{ss} & M_{sf} \\ \hline M_{fs} & M_{ff} + \Delta M_a \end{array} \right] \left\{ \begin{array}{c} \ddot{x}_s \\ \ddot{x}_f \end{array} \right\} + \left[\begin{array}{c|c} K_{ss} & K_{sf} \\ \hline K_{fs} & K_{ff} \end{array} \right] \left\{ \begin{array}{c} x_s \\ x_f \end{array} \right\} = 0 \quad (2.61)$$

$$\text{where } \left\{ \begin{array}{l} \Delta M_a = \begin{bmatrix} \ddots & & 0 \\ & \Delta m_i & \\ 0 & & \ddots \end{bmatrix} \\ \Delta m_i = \text{the lumped added mass (determined from Lewis forms,} \\ \text{for example) to be introduced to the} \\ \text{diagonals of the } i\text{'th degree of freedom.} \\ x_s = \text{degrees of freedom of the structure not in} \\ \text{contact with the fluid} \\ x_f = \text{degrees of freedom on the fluid-structure} \\ \text{interface} \end{array} \right.$$

The mode shapes, ϕ of equation 2.61 can be normalized such that

$$\left\{ \begin{array}{c} \phi_s \\ \phi_f \end{array} \right\}^T \left[\begin{array}{c|c} M_{ss} & M_{sf} \\ \hline M_{fs} & M_{ff} + \Delta M_a \end{array} \right] \left\{ \begin{array}{c} \phi_s \\ \phi_f \end{array} \right\} = \begin{bmatrix} \ddots & & 0 \\ & \tilde{M}_i & \\ 0 & & \ddots \end{bmatrix} = \tilde{\underline{M}} \quad (2.62)$$

and

$$\left\{ \begin{array}{c} \phi_s \\ \phi_f \end{array} \right\}^T \left[\begin{array}{c|c} K_{ss} & K_{sf} \\ \hline K_{fs} & K_{ff} \end{array} \right] \left\{ \begin{array}{c} \phi_s \\ \phi_f \end{array} \right\} = \begin{bmatrix} \ddots & & 0 \\ & \omega_i^2 \tilde{M}_i & \\ 0 & & \ddots \end{bmatrix} = \tilde{\underline{K}} \quad (2.63)$$

Then, instead of equation 2.61, the following is solved

$$\left[\begin{array}{c|c} M_{ss} & M_{sf} \\ \hline M_{fs} & M_{ff} + M_a \end{array} \right] \left\{ \begin{array}{c} \tilde{x}_s \\ \tilde{x}_f \end{array} \right\} + \left[\begin{array}{c|c} K_{ss} & K_{sf} \\ \hline K_{fs} & K_{ff} \end{array} \right] \left\{ \begin{array}{c} x_s \\ x_f \end{array} \right\} = 0 \quad (2.64)$$

where M_a is the added mass matrix calculated from either finite elements or boundary elements.

We then transform from global coordinates to modal coordinates using

$$\underline{x} = \sum_{i=1}^N \left\{ \frac{\phi_{s,i}}{\phi_{f,i}} \right\} q_i = \underline{\phi} \underline{q} \quad (2.65)$$

Then, multiplying equation 2.64 by $\underline{\phi}^T$ on the left hand side, we get

$$(\tilde{\underline{M}} + \tilde{\underline{M}}_a) \underline{\tilde{q}} + \tilde{\underline{K}} \underline{q} = 0 \quad (2.66)$$

$$\text{where } \underline{\tilde{M}}_a = \underline{\phi}^T \underline{M}_a \underline{\phi}$$

from which the final mode shape of the ship can be extracted from $\underline{q}$ using equation 2.65.

2.4.5 Modal Analysis Using Wet - Dry Mode Equivalence

The modal analysis approach assumes that ship mode shapes in water can be calculated by a combination of its mode shapes in air. However, if the assumption of wet and dry mode equivalence is made, modal analysis can be simplified significantly.

The governing equations of motion for a structure vibrating in air with no damping can be written as

$$\underline{M}_s \ddot{\underline{x}} + \underline{K}_s \underline{x} = 0, \quad (2.67)$$

where $\underline{M}_s$ is the structural mass matrix, $\underline{K}_s$ is the stiffness matrix and $\underline{x}$ is a vector of nodal displacements. If we assume sinusoidal motion with $\underline{x}(t) = \underline{X} \sin \omega t$, equation (2.67) reduces to

$$\{ \underline{K}_s - \omega^2 \underline{M}_s \} \underline{x} = 0. \quad (2.68)$$

Equation (2.68) is an eigenvalue problem for which the solution gives eigenvalues ω_a^2 and eigenvectors $\underline{\phi}_a$ of the structure vibrating in air. Further, the eigenvectors can be normalized such that

$$\begin{aligned} \underline{\phi}_a^T \underline{K}_s \underline{\phi}_a &= \omega_a^2 \\ \underline{\phi}_a^T \underline{M}_s \underline{\phi}_a &= 1 \end{aligned} \quad (2.69)$$

Next, consider the equations of motion for a structure vibrating in water with no damping. The fluid effects can be accounted for with an added mass matrix $\underline{M}_a$ such that

$$(\underline{M}_s + \underline{M}_a) \ddot{\underline{x}} + \underline{K}_s \underline{x} = 0. \quad (2.70)$$

Equation (2.70) is also an eigenvalue problem for the structure vibrating in water for which the solution gives eigenvalues ω_w^2 and eigenvectors $\underline{\phi}_w$. As

above, the eigenvectors can be normalized such that

$$\begin{aligned}\phi_w^T K_s \phi_w &= \omega_w^2 \\ \phi_w^T (M_s + M_a) \phi_w &= 1\end{aligned}\quad (2.71)$$

Hence substituting the eigenvectors into equation (2.70) we get

$$\{K_s - \omega_w^2 (M_s + M_a)\} \phi_w = 0.$$

Multiplying by ϕ_w^T ,

$$\phi_w^T \{K_s - \omega_w^2 (M_s + M_a)\} \phi_w = 0.$$

It has been shown from experimental and analytical results [7], [10] that the mode shapes for a structure vibrating in water is approximately the same as the mode shape in air. That is

$$\phi_a \approx \phi_w.$$

Thus

$$\phi_a^T \{K_s - \omega_w^2 (M_s + M_a)\} \phi_a = 0.$$

Multiplying the matrices (as per equation 2.71),

$$\omega_a^2 - \omega_w^2 \phi_a^T (M_s + M_a) \phi_a = 0.$$

Or,

$$\omega_a^2 - \omega_w^2 (1 + \phi_a^T M_a \phi_a) = 0.$$

And finally,

$$\frac{\omega_a^2}{\omega_w^2} = 1 + \phi_a^T M_a \phi_a. \quad (2.72)$$

Hence we find that the ratio of natural frequency in air to natural frequency in water can be found explicitly by equation (2.72).

This is the quickest approach of obtaining approximate results because only dry modes of vibrations are needed: re-analysis is not necessary. Natural frequencies of the ship in water are functions of the added mass matrix and of the dry mode shapes. By virtue of using equation (2.72), wet dry mode equivalence is assumed to be true. Note that this approach is identical to a *one mode* modal analysis.

Chapter 3

Finite Element Implementation of Added Mass

The purpose of this chapter is to illustrate how added mass, which was calculated from methods outlined in the previous chapter, can be implemented into an existing finite element program (VAST) [34]. A hierarchy of approaches, from the most time consuming to the least, will be presented.

There are many possible approaches of incorporating an added mass matrix in VAST. These generally fall within two categories: Wet Mode and Dry Mode. The Wet Mode category is defined as those approaches which incorporate the added mass matrix into the global mass matrix *before* natural frequencies and mode shapes are sought. The Dry Mode category is defined as those approaches which solve the equations of motion *without* the fluid and then incorporate the added mass to arrive at the final solution. It is important to remember that Dry Mode approaches require prior knowledge of structural behaviour in air before dynamic characteristics in fluid can be found. Below is a listing of the possible solution methods.

1. Wet Mode Category

- approximate both the fluid and structure with finite elements, to be referred to as VAST for the remainder of the report
- approximate the fluid with panel method (boundary elements) and structure with finite elements, to be referred to as PANEL

2. Dry Mode Category

- Finite Elements

- use VAST to find dynamic response in air and (for a specific mode of interest) find the lumped added mass (using Lewis formulae) and then solve for the response in water using VAST (VLV)

- Modal Superposition

- modal analysis using mode shapes in air and lumped masses (VLS)
- modal analysis using mode shapes in air and finite element added mass matrix (VFS)
- modal analysis using mode shapes in air and boundary element added mass matrix (VPS)

- Wet and Dry Mode Equivalence

- assume equivalence of modes in air and in water. This approach will be incorporated into both VFS and VPS. This approach is equivalent to modal analysis where only *one mode* is used in the approximation.

Discussions on each of the approaches will include: what are their advantages and disadvantages and how they can be implemented into VAST.

3.1 Wet Mode Category

Solution time for the Wet Mode category is generally much longer than those from the dry mode category. This is mainly because the bandwidth of the global mass matrix is much greater, due to the introduction of a full added mass matrix.

The simplest and most common of the Wet Mode category is the finite element method (VAST), where both the structure and the fluid are modelled using finite elements. This method gives accurate results for vertical modes of vibration and is already incorporated into the current version of VAST. This project will involve evaluating the numerical results of VAST and comparing them to experimental results.

A more elegant method in the Wet Mode category is to use boundary elements for the fluid domain (PANEL [32]). This method follows the derivations of Vorus [37] to arrive at an added mass matrix which is independent of vibration mode. This is deemed a more desirable approach because computation time required to calculate the added mass matrix is significantly less than the time used by the finite element approach. Figure 3.1 obtained from the report by Vernon [35] shows how PANEL can be implemented into VAST. The computer program PANEL will calculate an added mass matrix compatible with VAST. This project will compare boundary element results to experimental and finite element results.

3.2 Dry Mode Category

Unlike the Wet Mode approaches, Dry Mode approaches are relatively quick in solving the coupled fluid-structure interaction problem. A reduction of CPU time is achieved because these methods do not solve the submerged ship vibration problem. Vibration characteristics of the ship *in air* are first analysed with finite elements. Then, an added mass representative of the fluid surrounding the vibrating ship is calculated. This added mass can be either in the form of a lumped value (calculated for example by HYDRO) or a matrix (calculated by either VAST or PANEL). Finally, re-analysis of the structure with the added mass is done, from which mode shapes and natural frequencies of the ship *in water* is determined.

Re-analysis of the structure can be done using two different methods: full eigenvalue analysis, or modal analysis. It was been shown by Glenwright [10], [11] that for minimal structural property changes, modal analysis will give solutions much more quickly than full eigenvalue re-analysis. Hence, only the latter method will be fully explained in this report.

3.2.1 Modal Analysis

The method of modal analysis transforms a set of coupled equations (written in any user-defined coordinates) into a set of uncoupled equations in the principal coordinates. Each principal coordinate (or modal coordinate) describes a mode of vibration. The theory of modal analysis has been explained in the previous chapter. A more detailed report is given by Hutton and Baldwin

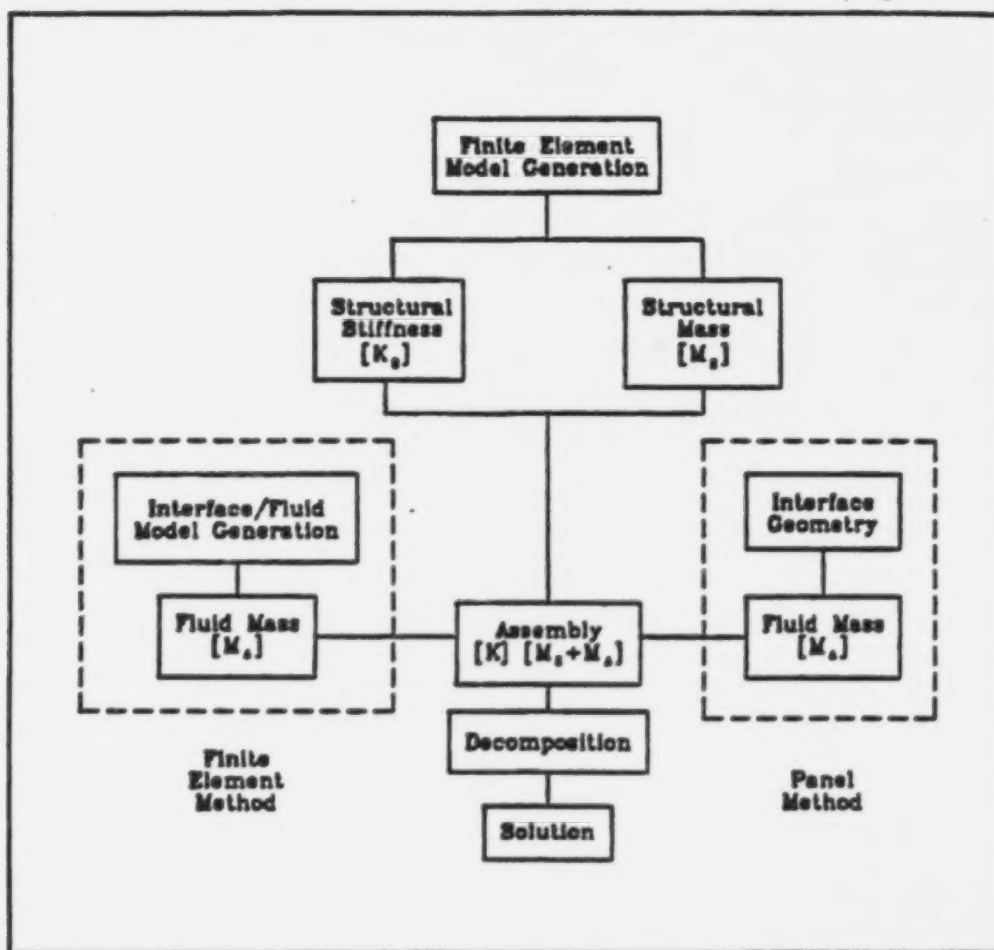


Figure 3.1: Stages of Evaluation for Ship Vibration Problems

[16]. Hutton and Baldwin wrote the computer program STRUM (for STRUctural Modification) which uses *modal condensation* to calculate the natural frequencies and mode shapes of modified structures using the mode shapes of the original structure. Modal condensation differs from modal analysis in that only a few mode shapes are used to uncouple the system of equations.

The original version of STRUM allowed the user to make modifications to the original (or baseline) structure. These modifications included changes in structural properties as well as additions of lumped added masses. However, STRUM cannot be used if added mass modifications are in the form of added mass matrices. Hence another computer program STRAM (for STRUctural modifications using Added Masses) was written to complement STRUM for situations where added mass modifications are in matrix form.

VLS

The VLS approach (which stands for VAST-Lumped mass-STRUM) combines the power of finite elements, the simplicity of boundary elements, and the elegance of modal analysis into a package which is relatively simple to use and yet provides a great deal of insight. First, natural frequencies and mode shapes of a structure in air are found using VAST. Then for a specific mode of interest, the effect of the fluid moving about the structure is found as a lumped mass using HYDRO. Finally, this mass is added to the original structure and the problem is re-analysed using STRUM, giving an approximate natural frequency for the mode of vibration which we have assumed initially.

The current version of HYDRO is fairly restrictive:

1. hull variation along the length of the ship is neglected,
2. the ship is restricted to only heave motions, and
3. cross section of the ship must be circular.

These restrictions were initially imposed to assess the validity of the approach using a simple problem. Constraints (2) and (3) can be easily resolved with minor changes to the computer code, but the resolution of constraint (1) will require a more significant re-coding. The elegance of the current version of HYDRO lies in its ability to analyse the fluid-structure interaction problem

in two dimensions (variation of velocity potential along the length is assumed to be negligible). If the analysis were extended to three dimensions, it would remain a lumped mass model, but CPU time would increase drastically. Since the idea behind VLS was to find a satisfactory result for a specific mode in the shortest amount of time, the three dimensional approach has not been pursued.

Currently, the executable version of VLS require the following input from the user

1. mode of interest,
2. node numbers to which the lumped masses are to be attached,
3. draft of the structure,
4. density of the fluid in which the structure is vibrating and,
5. radius of the cylindrical cross section.

VPS and VFS

When vibratory motion of the structure is complex, it appears that the more numerically rigorous methods would better represent the effect of the fluid than VLS. VPS (which stands for VAST-PANEL-STRUM) and VFS (which stands for VAST-FINITE ELEMENT-STRUM) use added mass matrices which account for *all* the displacements of the interface nodes. Application of VPS and VFS is identical to VLS, with the exception that STRAM is used instead of STRUM. The computer program STRAM is capable of running both VPS and VFS. Calculation of the added mass matrix as a function of nodal displacements has been shown in numerous papers [2], [28], and [37]. It is anticipated that VPS will be quicker to run, partly because the panel method added mass matrix can be found much faster. Note also that there are fewer degrees of freedom in the panel added mass. Hence finding $\phi^T M \phi$ will also take a shorter amount of time. While it is clear that VPS will take longer to solve than VLS, it should also be noted that it will give *all* the modes of vibration as compared to the *one* mode solution of VLS.

Chapter 4

Comparison of Results

The purpose of this chapter is to summarize results which were obtained using the different methods outlined in the previous chapter. The majority of the results presented are for the case of a ship model for which experimental results had been obtained.

4.1 Ship Model

This section compares experimental natural frequencies to predicted natural frequencies of a ship model and also compares the results of the different added mass modeling procedures.

4.1.1 Experimental Results

An experiment was conducted to study the accuracy of VAST [34] in determining free vibration response of a ship. The experimental model which was used is shown in figure 4.1. The model consisted of a long (96 inches) semi-circular (radius 4.196 inches) hull, reinforced with 11 bulkheads and two semi-spherical endcaps. Since the acrylic model is light and will not produce significant draft by itself, two steel blocks (with a combined weight of approximately twenty seven pounds) were placed inside the model to add extra draft. The properties of steel are well known (and can be easily modeled using finite elements), hence numerical results are expected to be close to the experimental results (shown in table 4.1). The final experimental model

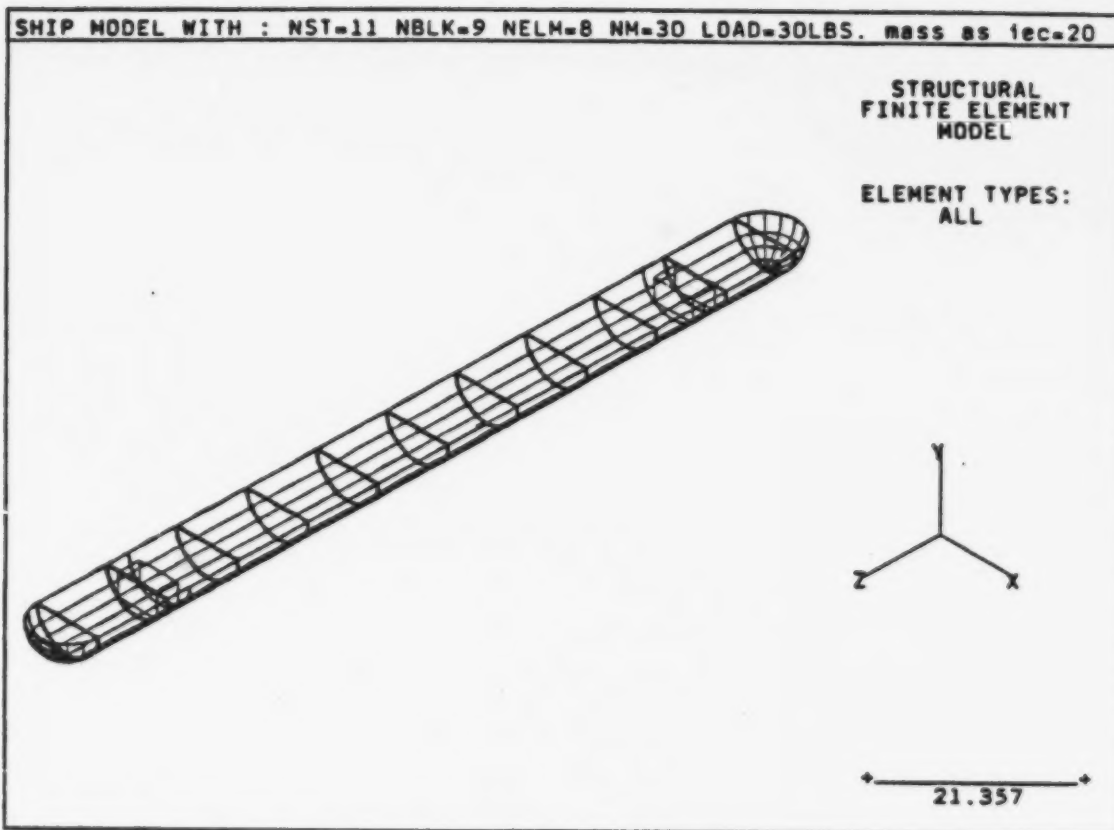


Figure 4.1: Experimental Ship Model

Mode	In Air	In Water
2 Node Vertical	20.63	8.75
3 NV	61.0	25.5
4 NV	109.5	47.0
5 NV	139.0	67.5
2 N Horizontal + 2 N Torsional	13.75	8.50
3 NH + 3NT	30.75	19.25
4 NH + 4NT	44.25	31.0
5 NH + 5NT	57.50	40.5

Table 4.1: Experimental Natural Frequencies (in Hertz) of Ship Model

Mode	Expt.	S20-8	Diff. (%)	S12-8	Diff. (%)
2 NV	20.63	20.36	-1.31	20.37	-1.26
3 NV	61.0	61.31	0.51	61.4	0.66
4 NV	109.5	108.6	-0.82	108.8	-0.64
5 NV	139.0	136.9	-1.51	137.8	-0.86
2 NH + 2NT	13.75	13.84	0.65	14.20	3.27
3 NH + 3NT	30.75	32.70	6.34	33.67	9.50
4 NH + 4NT	44.25	46.29	4.61	47.01	6.24
5 NH + 5NT	57.50	57.79	0.50	59.09	2.77

Table 4.2: Comparison of Natural Frequencies in Air - VAST

(including the steel blocks) draws approximately 2.1 inches of water.

4.1.2 Finite Element Results

Two finite element models were constructed to simulate the experimental ship model. The first model, S20-8, has 20 elements along the length and 8 elements around the circumference. The second model, S12-8 has 12 elements along the length and 8 elements around the circumference. S12-8 was created in hindsight because a complete finite element run (including fluid modeling) for S20-8 required more disk space than was available from the computer system available. Natural frequencies in air for the experiment and both finite element models are shown in table 4.2. As expected, results for free vibration in air of S20-8 were better than results of S12-8 because

Subroutine	S12-8	% of Total Time
$\underline{K}, \underline{M}$ Formation	0.45	4.06
Band Width Reduction	0.02	0.18
$\underline{K}, \underline{M}$ Assembly	0.08	0.72
$\underline{K}$ Addition	0.03	0.27
$\underline{M}$ Addition	0.02	0.18
Matrix Decomposition	0.95	8.57
Eigenvalue Analysis	9.53	86.02
Total	11.08	100.00%

Table 4.3: Comparison of CPU Times in Hours for S12-8 Using a VAX 750

there were more degrees of freedom in S20-8 than S12-8. It is interesting to note that the coupled torsional-horizontal modes are more sensitive to longitudinal modeling than the vertical modes, and the determination of accurate horizontal modes requires such a fine mesh. Natural frequencies for all the modes are calculated to within 10% of experimental results. Figures 4.2, 4.3, 4.4, and 4.5 show the vertical modes of vibration of the finite element model.

A summary of CPU times for S12-8 is given in table 4.3. Note that a significant portion (approximately 86%) of computer time was used in the eigenvalue analysis. CPU time for decomposition of mass and stiffness matrices into upper and lower triangular matrices represented at 8.57 percent of the total CPU time.

S20-8 was not utilized any further because of hardware constraints imposed by the computer system available. The model S12-8 was transformed into S12-F in which the water surrounding S12-8 was modeled using finite elements, thus the F suffix.

Water surrounding ship model S12-8 is approximated using two layers of 20-noded fluid finite elements. The outer boundaries of the fluid model extend 8.87 inches away and 9.45 inches below the center of the ship model. Recall that the ship has a radius of 4.2 inches and draws approximately 2.1 inches of water. The free surface is assumed to have zero velocity potential; that is, wave formation on the free surface has been neglected. This is a valid

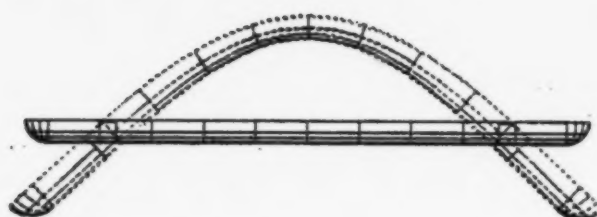


Figure 4.2: 2 Node Vertical Mode of Ship in Air

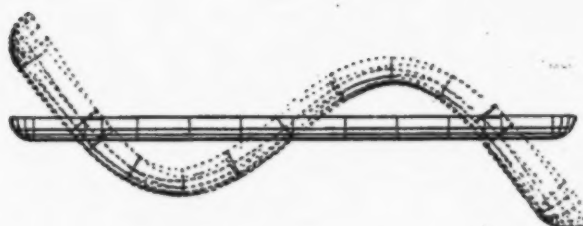


Figure 4.3: 3 Node Vertical Mode of Ship in Air



Figure 4.4: 4 Node Vertical Mode of Ship in Air



Figure 4.5: 5 Node Vertical Mode of Ship in Air

Mode	Expt.	S12-F	Diff. (%)
2 NV	8.75	8.76	0.1
3 NV	25.5	26.43	3.6
4 NV	47.0	47.94	2.0
5 NV	67.5	70.80	4.9
2 NH + 2 NT	8.5	8.84	4.0
3 NH + 3 NT	19.25	21.10	9.6
4 NH + 4 NT	31.0	33.09	6.7
5 NH + 5 NT	40.5	42.71	5.5

Table 4.4: Comparison of Natural Frequencies in Water - VAST

Mode	Δ Freq in Air	Δ Freq in Water	Difference
2 NV	-1.26	0.1	1.37
3 NV	0.66	3.6	3.66
4 NV	-0.64	2.0	2.64
5 NV	-0.86	4.9	5.76
2 NH + 2 NT	3.27	4.0	0.73
3 NH + 3 NT	9.50	9.6	0.10
4 NH + 4 NT	6.24	6.7	0.46
5 NH + 5 NT	2.77	5.5	2.73

Table 4.5: Comparison of Δ Frequency in Air and in Water

assumption since natural frequencies for all modes of interest, with the lowest being the 2 node vertical mode, are high enough such that wave formation effects can be neglected when compared to the fluid inertia effects. The outer most surfaces (including the radiation and sea bed boundaries) of the fluid domain have been given a zero velocity potential boundary condition.

Table 4.4 presents a comparison between the finite element and the experimental results and figures 4.6, 4.7, 4.8, and 4.9 show the vertical modes of vibration of the ship in water.

An indication of how *well* fluid finite elements modeled water surrounding the ship can be illustrated from the *difference* between predicted natural frequencies and experimental natural frequencies in water and in air. Table 4.5 shows quantitatively that the fluid finite element model has perhaps underestimated the extent of the fluid domain since the difference in predicted

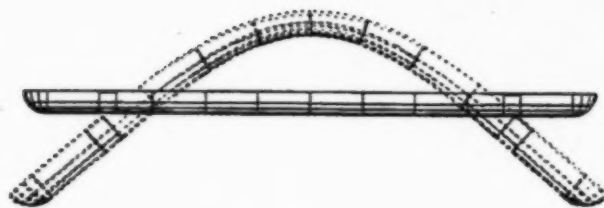


Figure 4.6: 2 Node Vertical Mode of Ship in Water

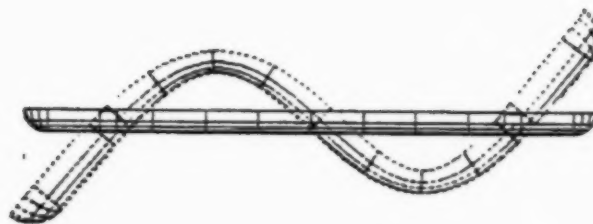


Figure 4.7: 3 Node Vertical Mode of Ship in Water

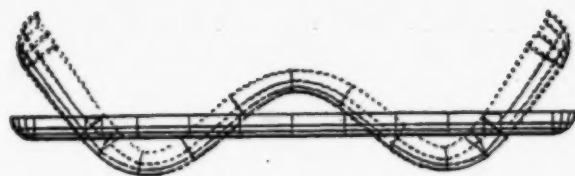


Figure 4.8: 4 Node Vertical Mode of Ship in Water

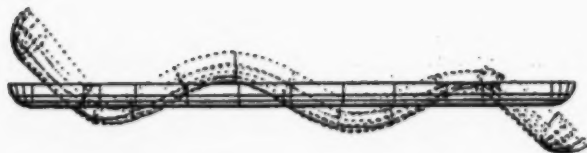


Figure 4.9: 5 Node Vertical Mode of Ship in Water

Subroutine	S12-F	% of Total Time
$\underline{K}, \underline{M}$ Formation	0.46	0.32
Band Width Reduction	0.02	0.01
$\underline{K}, \underline{M}$ Assembly	0.09	0.06
$\underline{K}$ Addition	0.03	0.02
$\underline{M}$ Addition	0.33	0.23
Matrix Decomposition	41.65	29.04
Eigenvalue Analysis	78.00	54.40
Fluid $\underline{M}$ Formation	0.23	0.16
Fluid $\underline{M}$ Assembly	1.10	0.77
Fluid $\underline{M}$ Decomposition	0.26	0.18
Fluid Added Matrix	21.24	14.81
Total	143.41	100%

Table 4.6: Comparison of CPU Times in Hours for Ship Model

natural frequency has increased from the air model to the water model. It is also interesting to note that the vertical modes of vibration have been affected more by fluid modeling than the coupled horizontal-torsional modes.

As suggested in the previous section, solution time for 12-F was quite large (see table 4.6). The breakdown of CPU times is significantly different than from table 4.3. The time for decomposition of $\underline{K}$ and $\underline{M}$ has increased from 0.95 hours to 41.65 hours, a four thousand percent increase. This is a result of a large increase in bandwidth of the global mass matrix due to the introduction of an added mass matrix. Formulation of the added mass matrix also represented a large portion of the total CPU time at more than 22 hours. Eigenvalue analysis was the most CPU intensive subroutine, increasing from 9.53 hours to 78 hours. Total time for analysis of S12-F was approximately 143 hours. Note that the time quoted is CPU time. Actual run time depends on how much the central processor is being shared with other users and is often double or even triple the CPU time. Thus the full finite element approach, even though it is fairly accurate, is very time consuming.

Mode	Experiment	S12-F	VLS	Diff (%)
2NV	8.75	8.76	7.16	-18.26
3NV	25.5	26.43	23.4	-11.46
4NV	47.0	47.94	42.2	-11.97
5NV	67.5	70.80	62.1	-12.29

Table 4.7: Comparison of Natural Frequencies in Water - VLS

4.1.3 Modal Analysis Results

It was established in the previous section that a full finite element solution of the ship model was excessively time consuming. A more practical alternative is to solve the structural vibration problem in water using modal analysis; in which ship behavior in air is used to predict its behavior in water.

Lumped Mass - VLS Approach

The VLS approach is an iterative procedure as shown in figure 4.10.

1. Dynamic characteristics of the structure in air are found using finite elements.
2. Fluid effects for a specific mode are quantified as a series of lumped masses.
3. Lumped masses are introduced into the structure as a mass modification.
4. The modified structure is re-analysed.
5. Steps 2 to 5 are repeated until convergence has been achieved. Note that this iteration will only be required if the added mass is frequency dependent.

Implementation of a lumped added mass into finite elements is straight forward; the mass is placed on the diagonal entry in the global mass matrix corresponding to the correct degree of freedom. Natural frequencies for the lower vertical beam modes are shown in table 4.7. Note that this approach is only valid for the determination of vertical modes.

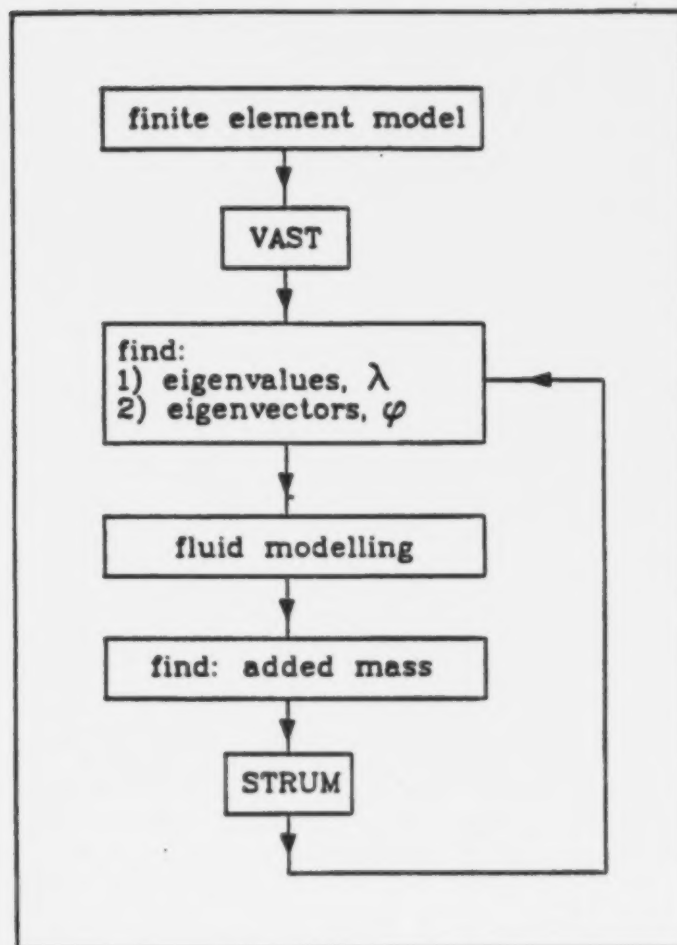


Figure 4.10: Iterative Solution

It was a surprise to see that VLS gave very poor results. However, closer examination of these results showed that the errors were a combination of three factors.

1. HYDRO has *over estimated* the fluid effects when compared to the other approaches used to calculate added mass. The added mass calculated by HYDRO was $0.243 \frac{\text{slugs}}{\text{ft}}$, which translated to a total added mass of 62.5 pounds. As a comparison, PNA [30] predicted the added mass to be $0.208 \frac{\text{slugs}}{\text{ft}}$ and VAST predicted the rigid body added mass to be $0.194 \frac{\text{slugs}}{\text{ft}}$, or a total added mass of 50 pounds. Thus the apparent mass of the ship (which is the sum of the structural mass and the added mass) calculated from HYDRO is 98 pounds as compared to 85.5 pounds from VAST: approximately 15 percent too high.
2. *J* factors have not been incorporated into the computer model. The added mass used is that derived assuming that the ship was heaving as a rigid body.
3. For the mass distribution existing in this particular case, the air mode shapes are *not* the same as water mode shapes. Figures 4.11, 4.12, 4.13, and 4.14 give a comparison of mode shapes (along the centerline of the ship model) in air versus mode shapes in water. As STRUM uses the air mode shapes to approximate the water modes shapes, this effect will be of significance.

Two finite element beam models were created to compare with the findings of VLS. The first beam model mimics the experimental ship model of this report. It is created such that its weight is 5.5 pounds, and when two 15 pound weights are placed at the correct locations, its 2 node vertical natural frequency is equivalent to the experimental value measured in air. This is achieved on a trial and error basis by adjusting Young's modulus of the beam model. Table 4.8 compares the experimental air frequencies with the simplified beam model air frequencies. The last column entry of table 4.8 is used to illustrate that there is a significant deviation (the largest being approximately 27%) between the experimental results and the simple beam model results. This indicates that a simple beam model can not satisfactorily predict all the lower vertical vibration modes for the model in question.

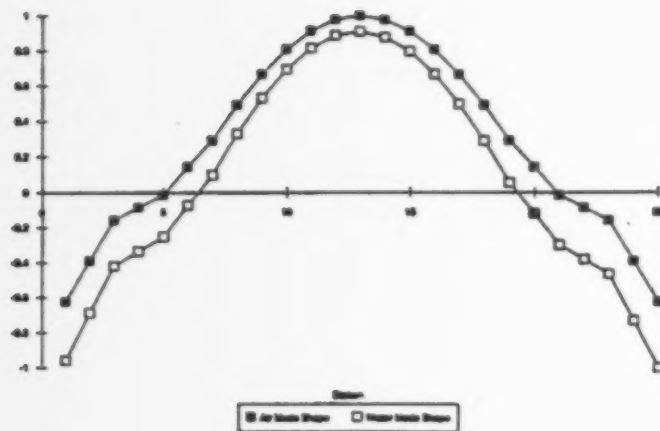


Figure 4.11: Comparison of Mode Shapes, 2 Node Vertical Mode

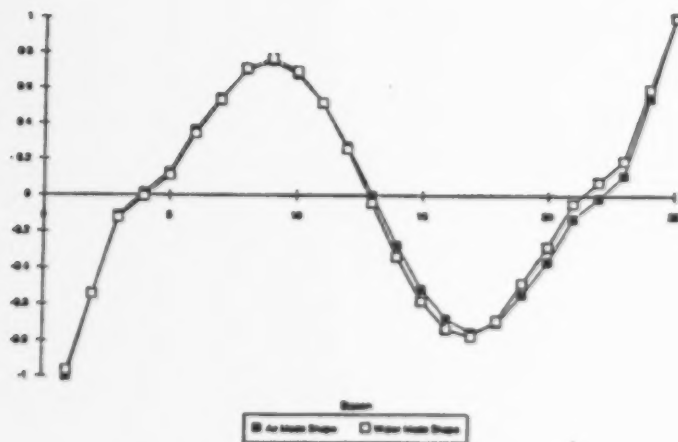


Figure 4.12: Comparison of Mode Shapes, 3 Node Vertical Mode

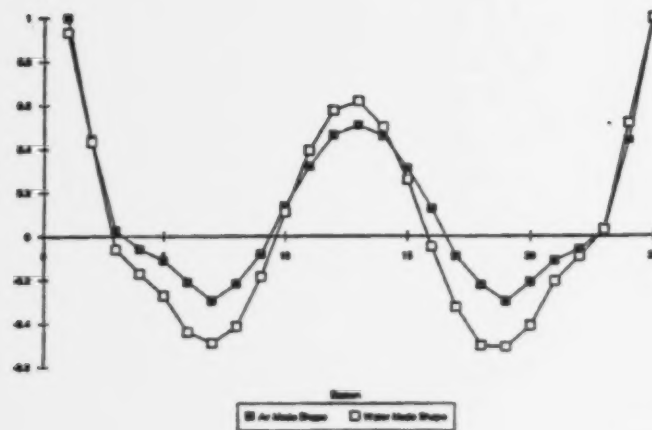


Figure 4.13: Comparison of Mode Shapes, 4 Node Vertical Mode

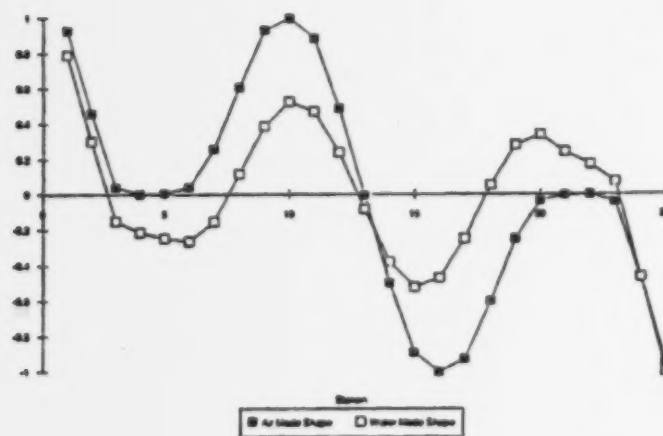


Figure 4.14: Comparison of Mode Shapes, 5 Node Vertical Mode

Mode	Expt. Air Freq	Beam Model Air Freq	Ratio
2NV	20.6	20.6	1.00
3NV	61.0	73.4	0.83
4NV	109.5	136.9	0.80
5NV	139.0	189.8	0.73

Table 4.8: Comparison of Air Frequencies Between Experimental Model and Simplified Beam Model

Mode	Glenwright's Expt. Air Freq	Beam Model Air Freq	Ratio
2NV	12.5	12.5	1.00
3NV	33.3	35.8	0.93
4NV	61.8	68.2	0.91
5NV	92.0	107.7	0.85

Table 4.9: Comparison of Air Frequencies Between Glenwright's Experimental Model and Simplified Beam Model

Once the in air case is simulated to satisfactory precision (for the two node vertical mode) the sectional added mass derived from HYDRO is then introduced to the beam model. VLS predicted the 2 node vertical mode to be 7.16 Hz whereas results from the simple beam model gave 7.01 Hz (recall that experiments showed the 2 node vertical mode in water to be 8.75 Hz). It is thus concluded that both methods have predicted natural frequencies which are significantly different than experimental results and the results of the beam model indicate that the added mass distribution used cannot be correct. It is recognized that employing a J factor of 0.9 which is appropriate for the two node vertical mode would increase the frequency but it would not increase it sufficiently to give rise to an accurate answer.

The second beam model approximates the experimental ship model in Glenwright's thesis [10], in which 30 pounds of weight are distributed evenly along the length of the model. The results tabulated in table 4.9 show that once again the beam model does not accurately represent the three node vertical and higher frequencies. However, in this case when the added mass distribution from HYDRO is added to the model, the 2 node vertical frequency equals 7.51 Hz which compares well with the measured water frequency of 7.75 Hz. From these examples it appears that adequate results

can be obtained for the case of a uniform structural mass distribution but for the case of discrete mass distribution used in the model ship the results are not accurate. The reason for this difference is not clear.

Added Mass Matrix - VFS and VPS Approach

It was established in the previous section that the VLS approach did not predict the natural frequency satisfactorily partly because the added mass effect was mis-represented. Thus, if fluid dynamics surrounding the ship is modeled more accurately, then the numerical results are expected to be much better. The next logical step to take would be to model the fluid domain in three dimensions. This section thus summarizes results which were obtained from using a fully coupled added mass matrix.

The modal analysis program STRUM, written to allow only lumped added mass changes to the global mass matrix, was re-written to include mass changes in matrix form. A new program STRAM (which stands for STRuctural modification using Added Mass matrices) was written to predict structural response in water, given the known structural properties in air and an added mass matrix. STRAM is a flexible program as it allows the user several options:

1. Use added mass matrix from finite elements (VFS); the V prefix designates that structural analysis in air is done with VAST and the S suffix designates that prediction of structural behavior is done with STRAM.
2. Use added mass matrix from panel method (VPS),
3. Allow specification of degree of complexity. Quick solutions can be obtained using single mode approximations. Complex analysis can be obtained using as many dry modes as the user desires.

The solution time for modal analysis increases as the number of modes used increases. Figure 4.15 shows the relationship between the time required to solve a problem with STRAM and the number of modes used for the analysis. Figure 4.15 was derived from a finite element model with 588 degrees of freedom. Clearly, this figure will not be the same for all finite element models. However, the *shape* of figure 4.15 will be the same for all

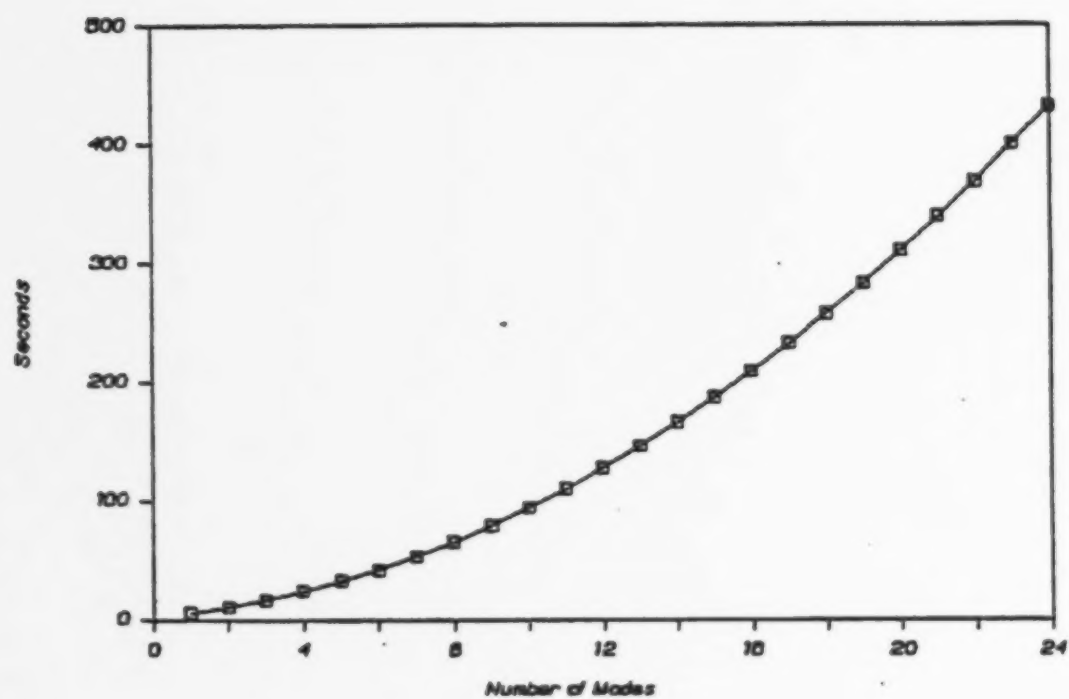


Figure 4.15: Comparison of Solution Times for Modal Analysis

Mode	Experiment	S12-F	VFS	Diff (%)
2NV	8.75	8.76	7.94	-9.4
3NV	25.5	26.43	25.78	-2.5
2NT + 2NH	8.5	8.84	8.49	-4.0
3NT + 3NH	19.25	21.10	20.91	-0.9
4NT + 4NH	31.0	33.09	33.36	0.8
5NT + 5NH	40.5	42.71	42.06	-1.5

Table 4.10: Comparison of Natural Frequencies - 14 Mode Approximation, S12-8

Mode	Experiment	S12-F	VFS	Diff (%)
2NV	8.75	3.76	7.95	-9.3
3NV	25.5	26.43	26.87	1.67
4NV	47.0	47.94	60.94	27.1
5NV	67.5	70.80	60.79	-14.1
2NT + 2NH	8.5	8.84	8.50	-3.9
3NT + 3NH	19.25	21.10	21.1	0.00
4NT + 4NH	31.0	33.09	33.43	1.0
5NT + 5NH	40.5	42.71	40.48	-5.2

Table 4.11: Comparison of Natural Frequencies - 1 Mode Approximation, S12-8

finite element models: the actual time will be a function of the size of the added mass matrix, as per equation 2.66.

Figure 4.15 illustrate that the time required to solve a twenty-mode problem is greater than solving 2 ten-mode problems. Physically this means that it takes longer to solve a 20 by 20 matrix than two 10 by 10 matrices. This result has significant implications. The user may choose to use single mode approximations instead of more complex analysis to save CPU time. The consequences of this approach will result in less accurate numerical results. Thus the user needs to find a compromise between quick solution times and accurate solutions. Results for the different options are tabulated in tables 4.10 and 4.11.

Results for VFS-14 (table 4.10) were encouraging. Natural frequencies for most modes have been calculated to within 4 percent of finite element

Mode	In Air Frequency (Hz)	Comment
8	14.2	2NT + 2NH
9	20.37	2NV
10	33.67	3NT + 3NH
11	47.01	4NT + 4NH
12	59.09	5NT + 5NH
13	61.4	3NV
18	108.8	4NV
20	137.8	5NV

Table 4.12: Natural Frequencies of S12-8, In Ascending Order

natural frequencies. The 4 node vertical and 5 node vertical modes were not included in the table because they could not be predicted by STRAM. In this case only the first 20 mode shapes of S12-8 were calculated by VAST. However, a floating ship is an unconstrained body, thus the first 6 mode shapes were rigid body modes with zero frequency. Since rigid body modes are not used, only 14 modes can be used by STRAM to approximate ship behavior. Table 4.12 lists the beam-like modes of S12-8 in ascending order. The 4 node and 5 node vertical modes represented the 18'th and 20'th modes respectively. It is possible that the eigenvalue routine was responsible for not being able to evaluate the response of these higher modes.

Further understanding of these results can be gained by looking at the orthogonality of the air mode shapes with respect to the added mass matrix. It is well known that mode shapes in air are orthogonal to the structural mass matrix; the same is true for water mode shapes with respect to the sum of the structural and added mass matrices. Mathematically, this can be written as

$$\phi_{ai}^T \underline{M}_s \phi_{aj} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (4.1)$$

and

$$\phi_{wi}^T (\underline{M}_s + \underline{M}_a) \phi_{wj} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (4.2)$$

However, air modes cannot be assumed to be orthogonal to the added mass matrix. That is,

$$\phi_{ai}^T \underline{M}_a \phi_{aj} \neq 0 \quad (4.3)$$

For a multi-mode analysis, the governing equation has been shown to be

$$(\underline{I} + \underline{\tilde{M}}_a) \ddot{\underline{q}} + \underline{\Omega} \dot{\underline{q}} = 0 \quad (4.4)$$

where

$$\begin{aligned} \underline{I} &= \text{identity matrix} \\ \underline{\tilde{M}}_a &= \underline{\phi}_a^T \underline{M}_a \underline{\phi}_a \\ &= \text{generalized added mass matrix} \\ \underline{\Omega} &= \text{matrix with } \omega_a^2 \text{ on diagonal entries} \end{aligned}$$

By inspection, this implies that if air mode shapes can be assumed to be orthogonal to the added mass matrix, a multi-mode analysis is identical to a one-mode analysis. Hence the off-diagonal entries of a generalized added mass matrix (with respect to the air modes) provide information as to the contribution of the air modes to the water modes. If off-diagonal terms are significant compared to diagonal terms, then a multi-mode analysis is required. However, if diagonal terms are larger than the off-diagonal terms, results for a multi-mode analysis will be similar to a one-mode analysis.

The symmetric matrix $\underline{\tilde{M}}_a$ is shown in figure 4.16. Note that, for identification purposes, boxes are drawn on the diagonal degrees of freedom representing vertical vibration modes. $\underline{\tilde{M}}_a$ is a 14 by 14 matrix, with each degree of freedom representing a specific vibration mode in air. A summary of d.o.f. against global vibration mode in air is given in table 4.13.

CPU time for VFS-14 was 4 hours 39 minutes. A complete finite element analysis of S12-F took approximately 143 hours whereas CPU time for modal analysis is approximately 38 hours (11 hours for the structural vibration problem, 22 hours for added mass matrix formulation, and 4.65 hours for modal analysis). This represents a significant saving in cost.

The results for a 14-mode approximation are presented in table 4.10 and for a 1-mode approximation in table 4.11. The results for the vertical modes in table 4.11 are surprisingly poor. It was first suspected that the computer program STRAM gave incorrect results. A simple check of the program can be done by looking at the equations of motion closely. Recall that the natural frequency of a structure vibrating in air is given as

$$\omega_a^2 = \frac{\underline{\phi}_a^T \underline{K}_s \underline{\phi}_a}{\underline{\phi}_a^T \underline{M}_s \underline{\phi}_a}$$

$$\bar{M}_a = \begin{bmatrix} 0.181 & -0.006 & 0.000 & -0.314 & 0.030 & -0.364 & -0.034 & 0.004 & -0.006 & -0.084 & -0.021 & -0.001 & 0.173 & 0.007 \\ 1.795 & -0.007 & -0.021 & 0.165 & 0.039 & 0.001 & 0.070 & 0.052 & 0.425 & 0.002 & -0.002 & -0.008 & 0.000 \\ \boxed{5.564} & 0.000 & 0.002 & -0.014 & 0.179 & 0.000 & 0.079 & -0.351 & -1.100 & 0.498 & 0.011 & 0.150 \\ 1.548 & 0.086 & 0.490 & 0.042 & 0.018 & 0.004 & -0.019 & -0.012 & 0.000 & -0.035 & 0.002 \\ 0.977 & -0.108 & 0.006 & 0.119 & 0.052 & -0.074 & 0.005 & -0.007 & 0.021 & 0.012 \\ 1.131 & -0.347 & -0.019 & 0.094 & 0.212 & -0.041 & 0.022 & 0.222 & -0.161 \\ \boxed{4.215} & -0.003 & -1.047 & -0.378 & 0.042 & -0.390 & -0.059 & 2.086 \\ 0.032 & 0.000 & 0.056 & 0.000 & 0.000 & 0.006 & 0.000 \\ 2.196 & 0.174 & -0.003 & 0.163 & 0.021 & -1.391 \\ 1.003 & -0.024 & 0.088 & 0.121 & 0.770 \\ 0.812 & -0.419 & 0.015 & 0.115 \\ \boxed{2.186} & 0.004 & 0.248 \\ 1.439 & -0.086 \\ \boxed{4.140} \end{bmatrix}$$

Figure 4.16: Generalized Added Mass for S12-8, $\bar{M}_a$

D.O.F.	Mode
1	1NT
2	2NT + 2NH
3	2NV
4	3NT + 3NH
5	4NT + 4NH
6	5NT + 5NH
7	3NV
12	4NV
14	5NV

Table 4.13: Summary of Degrees of Freedom in Generalized Added Mass for S12-8

from which the mode shapes are normalized such that

$$\phi_a^T K_s \phi_a = \omega_a^2$$

and

$$\phi_a^T M_s \phi_a = 1$$

Moreover, the natural frequency of a structure vibrating in water is given as

$$\omega_w^2 = \frac{\phi_w^T K_s \phi_w}{\phi_w^T (M_s + M_a) \phi_w}$$

Hence

$$\frac{\omega_a^2}{\omega_w^2} = \frac{\phi_a^T K_s \phi_a}{\phi_w^T K_s \phi_w} \frac{\phi_w^T (M_s + M_a) \phi_w}{\phi_a^T M_s \phi_a} \quad (4.5)$$

Note that equation 4.5 is an exact correlation; no assumptions have been made. This correlation can be used as a check on the numerical results of STRAM as well as on the errors inherent to the assumption of equivalence of modes.

Numerical results from STRAM can be verified by setting

$$\phi_a \approx \phi_w$$

Mode	ω_a (Hz)	ω_w (Hz)	LHS
2NV	20.37	7.95	2.562
3NV	61.4	26.87	2.285
4NV	108.8	60.94	1.785
5NV	137.8	60.79	2.267
2NT + 2NH	14.20	8.50	1.671
3NT + 3NH	33.67	21.1	1.576
4NT + 4NH	47.01	33.43	1.406
5NT + 5NH	59.09	40.48	1.460

Table 4.14: Ratio of Natural Frequencies

Thus equation 4.5 becomes

$$\frac{\omega_a}{\omega_w} \approx \sqrt{\frac{\phi_a^T (\underline{M}_s + \underline{M}_a) \phi_a}{\phi_a^T \underline{M}_s \phi_a}} \approx \sqrt{1 + \phi_a^T \underline{M}_a \phi_a} \quad (4.6)$$

since the mode shapes have been normalized such that

$$\phi_a^T \underline{M}_s \phi_a = 1$$

A computer program, independent of STRAM, was written to find the value of $\phi_a^T \underline{M}_a \phi_a$. A check on the results of STRAM can be done by comparing the right hand side of equation 4.6 to the left hand side. Note that in the comparison, ω_w is the frequency given by the single-mode analysis and $\phi_a^T \underline{M}_a \phi_a$ is calculated by the independent program. The results are shown in tables 4.14 and 4.15. Comparisons of the LHS to the RHS show that they are indeed equivalent, which imply that VFS-1 results are correct (under the assumption of mode equivalence). Hence the differences between VAST and STRAM are not caused by numerical errors.

Equation 4.6 can also be used to provide an indication of the magnitude of errors introduced by assuming equivalence between wet and dry modes. The natural frequency of any structure can be calculated by the ratio of its potential energy (or generalized stiffness) to its kinetic energy (or generalized mass). Hence, a comparison of how well the air modes approximate the water

Mode	$\phi_a^T \underline{M}_a \phi_a$	RHS
2NV	5.564	2.562
3NV	4.215	2.284
4NV	2.186	1.785
5NV	4.140	2.267
2NT + 2NH	1.795	1.672
3NT + 3NH	1.548	1.596
4NT + 4NH	0.977	1.406
5NT + 5NH	1.131	1.460

Table 4.15: Ratio of Natural Frequencies Calculated from Generalized Added Mass

modes can only be done if the mode shapes are normalized to the same factor. For example, each mode shape in air have been normalized by VAST such that

$$\phi_a^T \underline{K}_a \phi_a = \omega_a^2$$

and

$$\phi_a^T \underline{M}_a \phi_a = 1$$

Similarly, each mode shape in water have been normalized by VAST such that

$$\phi_w^T \underline{K}_w \phi_w = \omega_w^2$$

and

$$\phi_w^T (\underline{M}_s + \underline{M}_a) \phi_w = 1$$

If the mode shapes in water are normalized such that

$$\phi_w^T \underline{K}_w \phi_w = \omega_a^2$$

and hence

$$\phi_w^T (\underline{M}_s + \underline{M}_a) \phi_w = \frac{\omega_a^2}{\omega_w^2}$$

The mode shapes ϕ_a and ϕ_w are now normalized to the same factor. Comparisons between $\phi_w^T (\underline{M}_s + \underline{M}_a) \phi_w$ and $\phi_a^T (\underline{M}_s + \underline{M}_a) \phi_a$ will show how well the mode shapes in air have approximated the kinetic energy of the ship in water. Results of the comparison are shown in tables 4.16 and 4.17.

Mode	$\phi_w^T (\underline{M}_s + \underline{M}_a) \phi_w$
2NV	5.407
3NV	5.397
4NV	5.151
5NV	3.788
2NT + 2NH	2.580
3NT + 3NH	2.546
4NT + 4NH	2.018
5NT + 5NH	1.914

Table 4.16: Generalized Added Mass Calculated from Mode Shapes in Water

Mode	$\phi_a^T (\underline{M}_s + \underline{M}_a) \phi_a$
2NV	6.564
3NV	5.215
4NV	3.186
5NV	5.140
2NT + 2NH	2.795
3NT + 3NH	2.548
4NT + 4NH	1.977
5NT + 5NH	2.131

Table 4.17: Generalized Added Mass Calculated from Mode Shapes in Air

Note that ω_w for this comparison is the natural frequency in water calculated by VAST.

Tables 4.16 and 4.17 show that air mode shapes did not approximate water mode shapes as well as expected. The differences can be attributed to the fact that, for this model, mode shapes in air are different to the mode shapes in water (as shown in figures 4.11, 4.12, 4.13, and 4.14). Furthermore, the difference in mode shapes can be attributed to the fact that weight distribution of the ship is different in air than in water. The apparent kinetic energy of the ship is mis-represented by assuming mode equivalence, as shown in tables 4.16 and 4.17.

An additional step was performed to check the validity of numerical results from STRAM. This step involved creating a ship model similar to that of Glenwright's thesis [10] and, using the air modes from that model, estimat-

Mode	Glenwright's Expt.	VFS-1
2NV	12.5	11.93
3NV	33.25	32.81
4NV	61.75	56.98

Table 4.18: Comparison of Air Natural Frequencies (Hz) for Ship Model With Uniform Mass Distribution

Mode	Glenwright's Expt.	VFS-1
2NV	7.75	7.93
3NV	21.25	21.88
4NV	40.75	42.0

Table 4.19: Comparison of Water Natural Frequencies (Hz) For Ship Model With Uniform Mass Distribution

ing the natural frequencies in water and comparing them with Glenwright's experimental results. Glenwright's experimental model, with 30 pounds of weight uniformly distributed along the length of the hull is more representative of a real ship than the model with the point masses. The results, shown in tables 4.18 and 4.19, indicate that STRAM is capable in this case of accurately predicting structural natural frequencies in water. The coupled horizontal torsional modes were not included in the tables because, in order to minimize computation time and computer resource requirements, the uniform weight distribution was modeled as point loads: rotational inertia was not modeled.

4.1.4 PANEL Results

The PANEL method, which uses boundary element theory, serves as an alternative to the conventional finite element method, in determining the added mass matrix. Derivation of the added mass matrix is much faster with boundary elements. Approximately 15 minutes of interactive run time was required to generate an added mass matrix as compared to 22 hours CPU time for S12-8 (fluid finite elements).

A full finite element solution with PANEL added mass could not be obtained due to hardware constraints. However, a comparison was made be-

Mode	Experiment	VFS-1	VPS-1	Diff (%)
2NV	8.75	7.95	6.86	-13.7
3NV	25.5	26.87	23.17	-14.0
4NV	47.0	60.94	51.49	-15.5
5NV	67.5	60.79	53.21	-12.5
2NT + 2NH	8.5	8.50	7.87	-7.4
3NT + 3NH	19.25	21.1	19.38	-8.2
4NT + 4NH	31.0	33.43	30.20	-8.7
5NT + 5NH	40.5	40.48	36.97	-8.7

Table 4.20: Comparison of Natural Frequencies - VPS, 1 Mode Approximation

tween VPS and VFS (using identical fluid meshes) to determine how close the PANEL added mass matrix is to the finite element added mass matrix. Results for this comparison are shown in table 4.20. The vertical modes were calculated to within approximately 15 percent and the coupled horizontal-torsional modes to within 9 percent.

A simple manipulation of added mass matrices will indicate how poorly PANEL compares to VAST. For rigid body heave motion of the ship model, the total added mass is the sum of y d.o.f. added mass entries for all wetted nodes. Hence a simple computer program was written to extract all y d.o.f. entries from both added mass matrices. Heave added mass for boundary elements was 0.195 (or 75.3 pounds) compared to 0.129 (or 49.8 pounds) for finite elements. The apparent mass of the ship is 110.9 pounds for PANEL method model and 85.4 pounds for finite element method model. The panel method has over predicted the added mass by almost thirty percent. This is consistent with results from VPS-1, where heave natural frequencies are approximately fifteen percent too low.

It is not clear why the Panel results perform so poorly. It would have been expected that they would produce the most accurate representation of the fluid added mass.

Subroutine	VAST	Modal Analysis
$\underline{K}, \underline{M}$ Formation	106.17	106.17
$\underline{K}, \underline{M}$ Assembly	16.15	16.15
$\underline{K}$ Addition	8.41	8.41
$\underline{M}$ Addition	12.82	5.33
Matrix Decomposition	66.33	66.54
Eigenvalue Analysis	1429.22	1167.54
Fluid $\underline{M}$ Formation	117.84	117.84
Fluid $\underline{M}$ Assembly	37.54	37.54
Fluid $\underline{M}$ Decomposition	53.10	53.10
Fluid Added Matrix	80.45	80.45
STRAM		207.25
Total	1928.08	1866.32

Table 4.21: Comparison of CPU Times in Seconds for Cantilevered Plate

4.2 Plate Model

4.2.1 Cantilevered Boundary Condition

Another finite element model, with less degrees of freedom than S12-8, was formed to further test STRAM. The model is a steel plate 8 in. by 8 in. and 0.105 in. thick fully immersed in fluid, see figure 4.17. Figure 4.18 shows a comparison between finite element results and STRAM results. STRAM gave relatively good results. The first 19 of 20 modes calculated were within 8% accuracy. Note that 18 out of 19 modes were calculated to less than 4%. Good results were obtained because mode shapes in air are the same as mode shapes in water. This can be attributed to the fact that properties of the plate (i.e. weight distribution) are the same in air as in water.

Table 4.21 gives a breakdown of CPU times for finite elements and STRAM. The only significant difference between finite elements and STRAM is in the amount of time for eigenvalue analysis: a savings of 61.76 seconds is made.

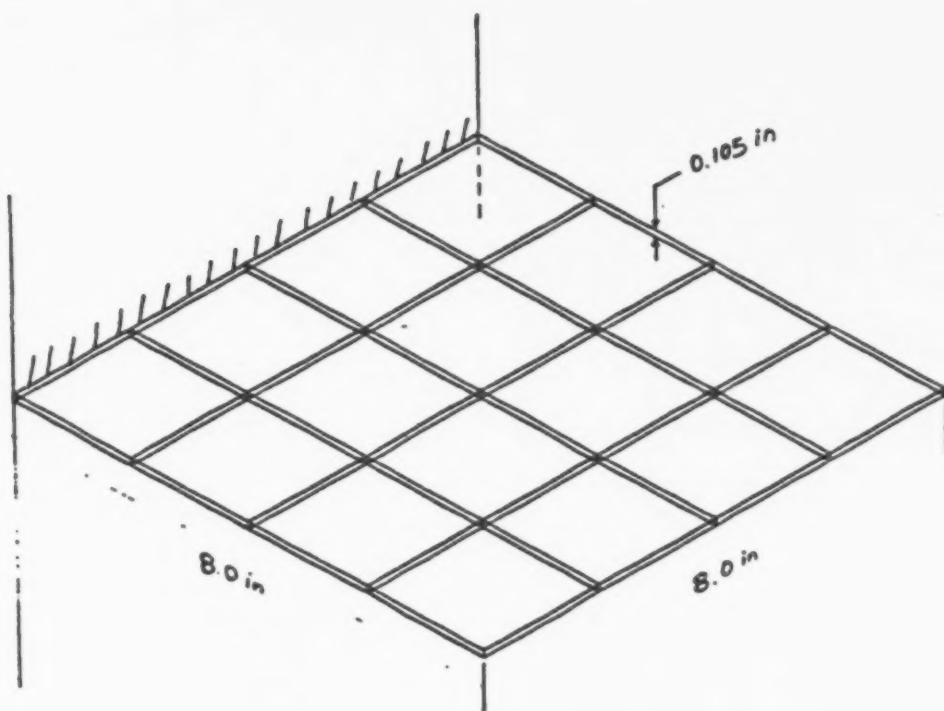


Figure 4.17: Plate Model

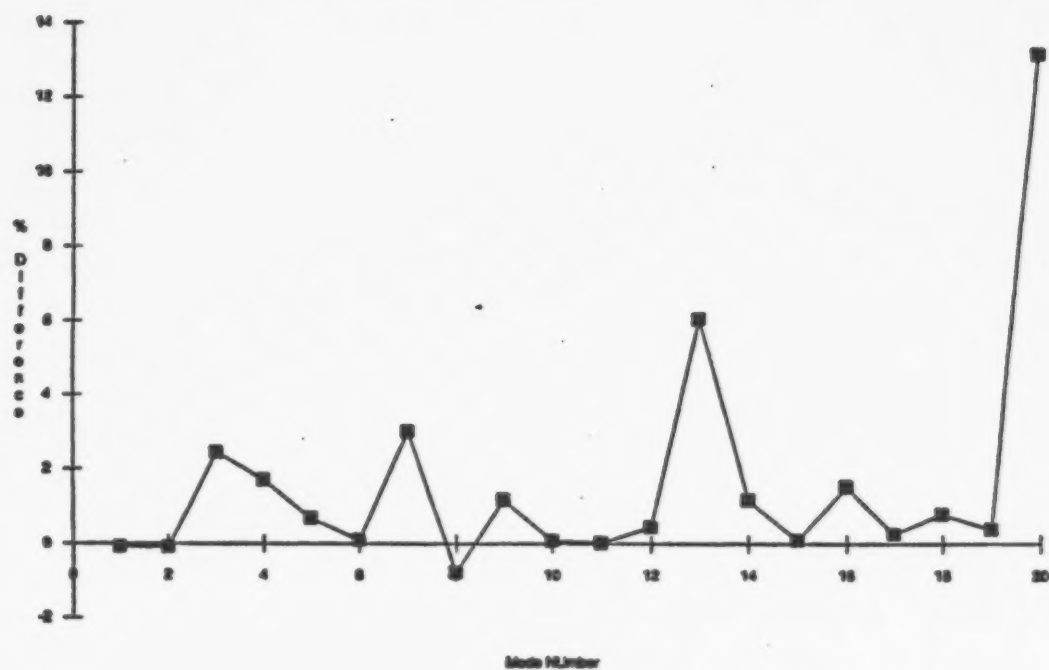


Figure 4.18: Cantilevered Plate - Comparison of Results Between Finite Element Analysis and Modal Analysis

4.2.2 Free-Free Boundary Condition

A free-free plate similar to the cantilevered plate, with the exception of the boundary conditions, was modeled by VAST. This model has 45 degrees of freedom more than the cantilevered plate, 9 nodes along the previously cantilevered edge with 5 d.o.f. per node. Results for the free free plate are shown in figure 4.19.

Table 4.22 gives a breakdown of CPU times for the free free plate. STRAM seems more attractive in this example because a savings of more than one thousand seconds is achieved, compared to approximately 60 in the cantilevered plate model. This is a combination of several factors.

1. The free free plate is a *free free* model. The stiffness matrix is singular. Recall that the system of equations for a free vibration problem is given by

$$\underline{M} \ddot{\underline{x}} + \underline{K} \underline{x} = 0$$

From which the solution is found from

$$\underline{K} \underline{x} = \omega^2 \underline{M} \underline{x}$$

If the structure is unconstrained (i.e. a free free system) the determinant of the system of equations cannot be found. Thus to solve this problem, a matrix proportional to the mass matrix is added to both sides of the equation, or

$$(\underline{K} + \alpha \underline{M}) \underline{x} = (\omega^2 + \alpha) \underline{M} \underline{x}$$

Hence more CPU time is required to decompose the matrices.

2. The free free plate has an extra 45 degrees of freedom. Studies [42] showed that CPU time required to solve eigenvalue problems is proportional to the square of the size problem. Hence more CPU time is dedicated to solve the eigenvalue problem.

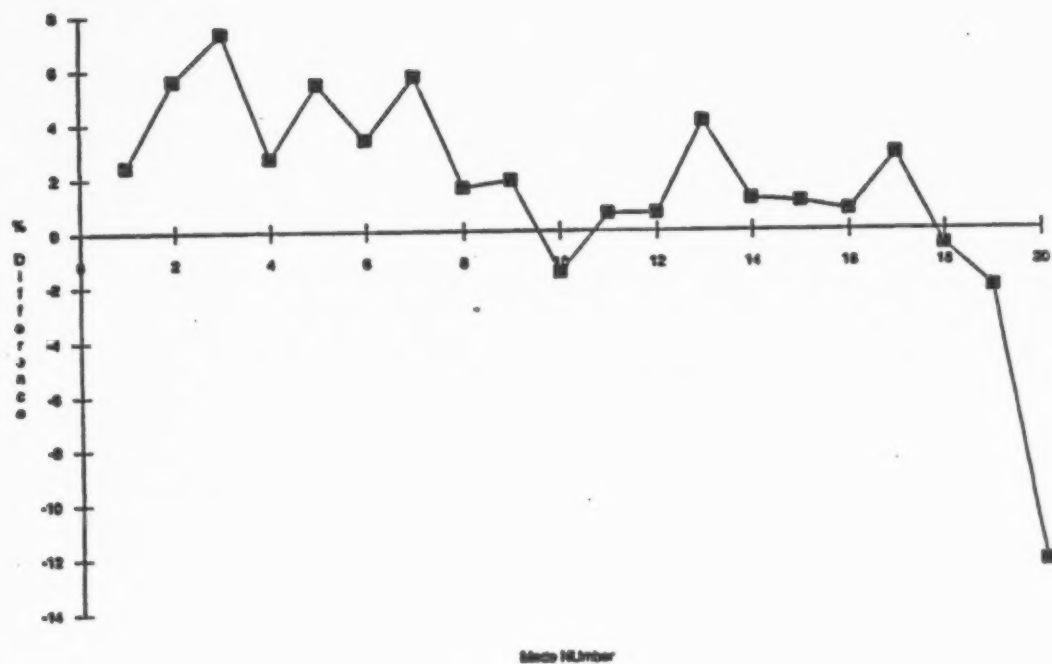


Figure 4.19: Free Free Plate – Comparison of Results Between Finite Element Analysis and Modal Analysis

Subroutine	VAST	Modal Analysis
$\underline{K}, \underline{M}$ Formation	107.03	107.03
$\underline{K}, \underline{M}$ Assembly	16.66	16.66
$\underline{K}$ Addition	8.94	8.94
$\underline{M}$ Addition	12.57	5.80
Matrix Decomposition	121.45	74.52
Eigenvalue Analysis	2321.53	1208.78
Fluid $\underline{M}$ Formation	117.22	117.22
Fluid $\underline{M}$ Assembly	37.62	37.62
Fluid $\underline{M}$ Decomposition	53.72	53.72
Fluid Added Matrix	80.48	80.48
STRAM		151.33
Total	2877.22	1862.10

Table 4.22: Comparison of CPU Times in Seconds for Free Free Plate

Chapter 5

Conclusions

1. In predicting the effects of added mass on the natural modes of ship structures, the response in the vertical modes should be treated separately from the coupled horizontal torsional modes.
2. In predicting the vertical mode responses, it is practical and efficient to model the effect of the fluid as point masses distributed along the length of the hull model. Such models can be analyzed efficiently by the use of modal condensation as, for example, when using the program STRUM.

The added mass values can theoretically be obtained by using either Lewis formula or the Panel method. However, in the present investigation the added mass values computed by these two methods only agreed to within 17 percent for the rigid body mode, indicating a possible problem with the Panel results.

3. In predicting the coupled horizontal torsional modes a full added mass matrix is required in order to correctly model the effects of the fluid. This mass matrix is most efficiently obtained using the Panel method, and the effect of incorporating the matrix can be most efficiently handled using STRUM.
4. The results presented in the report for a ship model with a significant discontinuity in its loading distribution shows that in such cases the air mode shapes and the water mode shapes are significantly different and that difference will have a significant affect on the ability of STRUM

or any modal analysis based procedure to accurately predict the effect of the added mass.

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This report presents a comparison of added mass modeling techniques that may be used to determine the vibration response characteristics of ships in water. The mathematical treatment of added mass is reviewed, and a number of numerical approaches are discussed. Examples are presented of the different numerical approaches when applied to a ship model and numerical results for the natural frequencies are compared with experimentally determined values.

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